



# ADAPTIVE, HIGH-ORDER SIMULATIONS OF LARGE-SCALE COMPLEX FLOWS

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## ABSTRACT

A new class of time discretizations based on the *exponential time integration* is successfully developed to speed up high-order computations of real-world fluid dynamics problems with *hp* adaptation. The resulted *HA3D* solver: an adaptive order discontinuous Galerkin (DG) code for arbitrarily shaped elements is presented including various state-of-art methods, aiming to develop an extreme-capability CFD flow solver for predicting increasingly challenging fluid dynamics problems of real world.

## NUMERICAL METHODS

### Time discretizations

1. Implicit methods
  - (a) Rosenbrock Runge-Kutta ( $s = 2$ )
  - (b) Pointwise Jacobi iteration
  - (c) Lower-Upper Symmetric Gauss Seidel (LU-SGS)
  - (d) Symmetric Gauss Seidel (SGS)
  - (e) Newton-Krylov: *ILU*-GMRES(m)
  - (f) Dual time stepping
2. Explicit Runge Kutta (SSP, Jameson)
3. PCEXP scheme for unsteady flows (*S.-J. Li et al. JCP, 365(2018) 206-225*)
4. EXP1 scheme for steady flows (*S.-J. Li et al. JCP, 365(2018) 206-225*)
5. Exponential  $p$ -multigrid method: eMG (*S.-J. Li, arXiv:1807.01151*)
6.  $p$ -adaptive global coupling methods: *S.-J. Li, AIAA 2020 (to be appeared)*

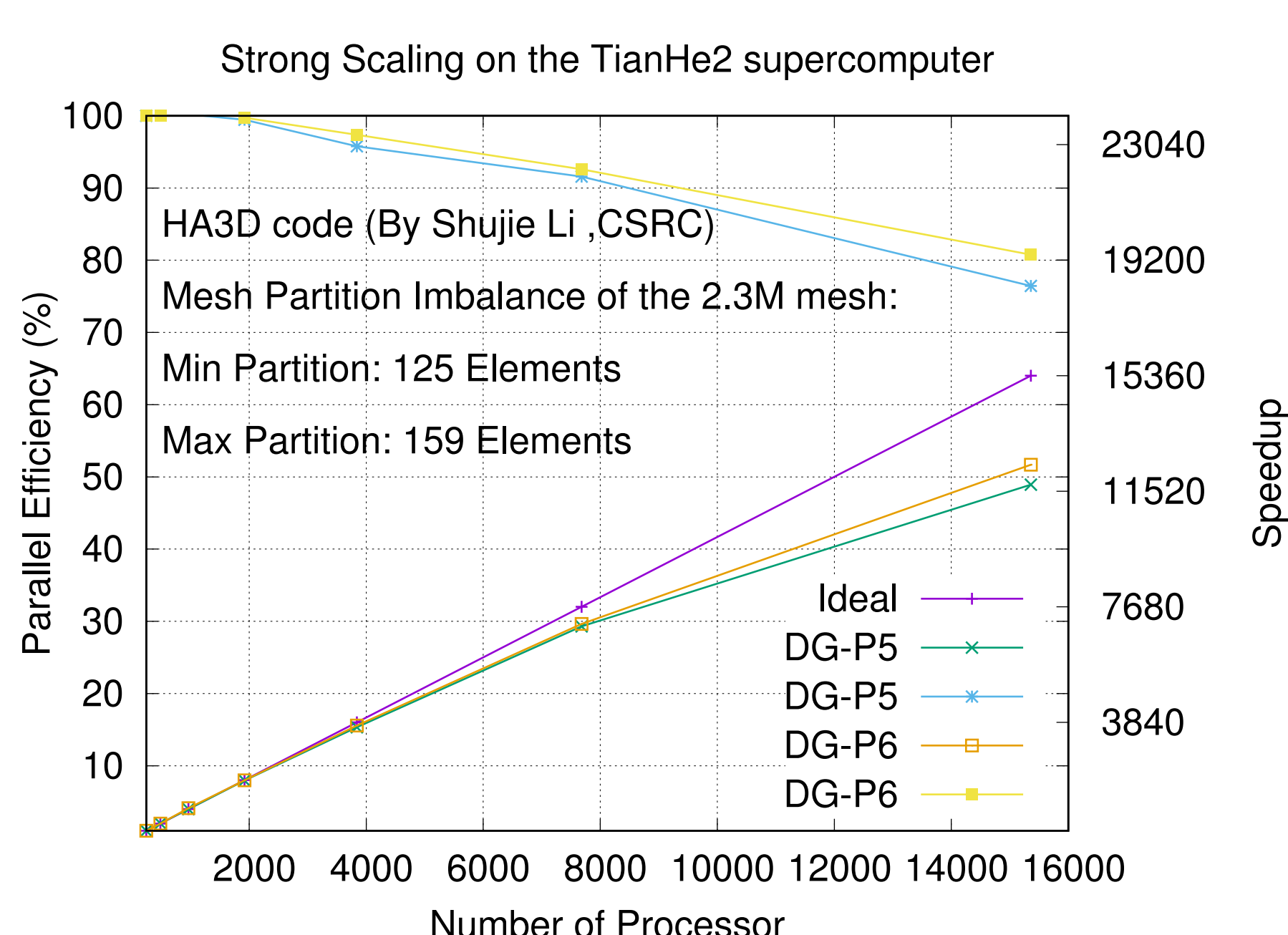
### Spatial discretizations

1. Discontinuous Galerkin up to 11-th order
2. Kinetic fluxes & rotating Riemann solver
3. Adaptive  $p$ -multigrid
4. Rotor source term

### Multibody Moving Framework

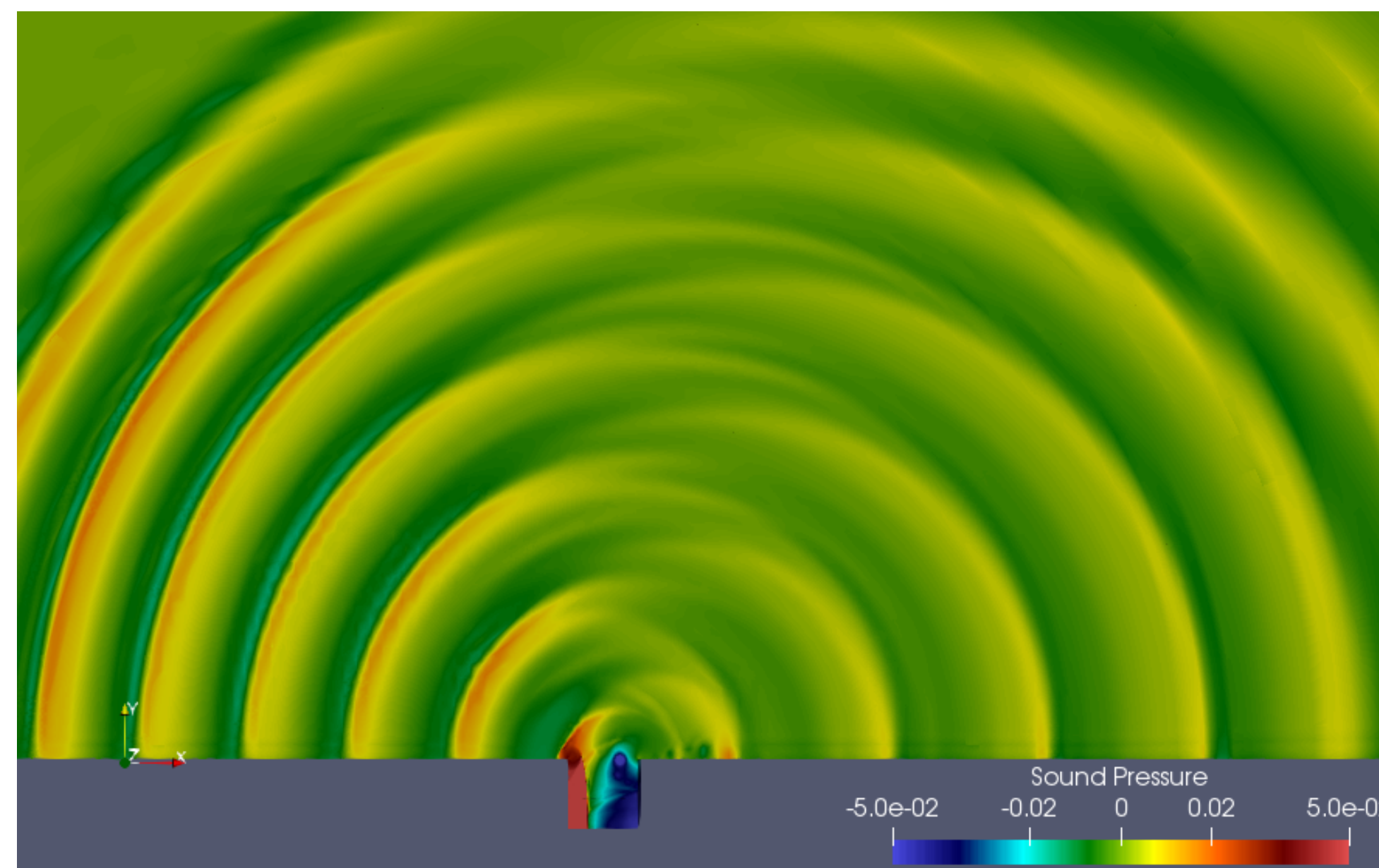
1. Arbitrary Lagrangian-Eulerian (ALE): single and multiple reference frames

## PARALLEL SCALABILITY



Strong scaling performance on TIANHE2 supercomputer at Guangzhou, China.

## FAST EXPONENTIAL TIME MARCHING FOR 3-D FLUID DYNAMICS



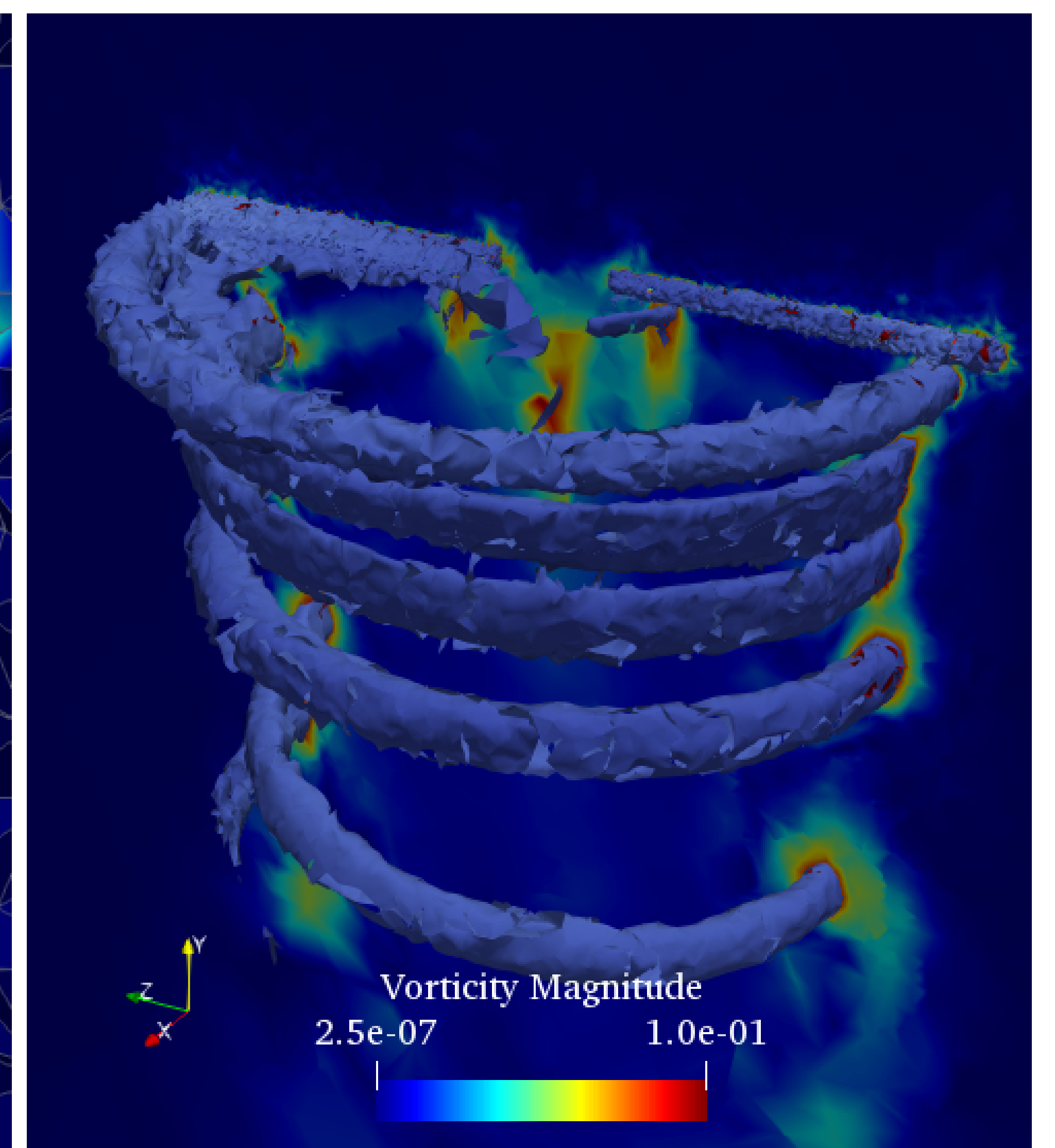
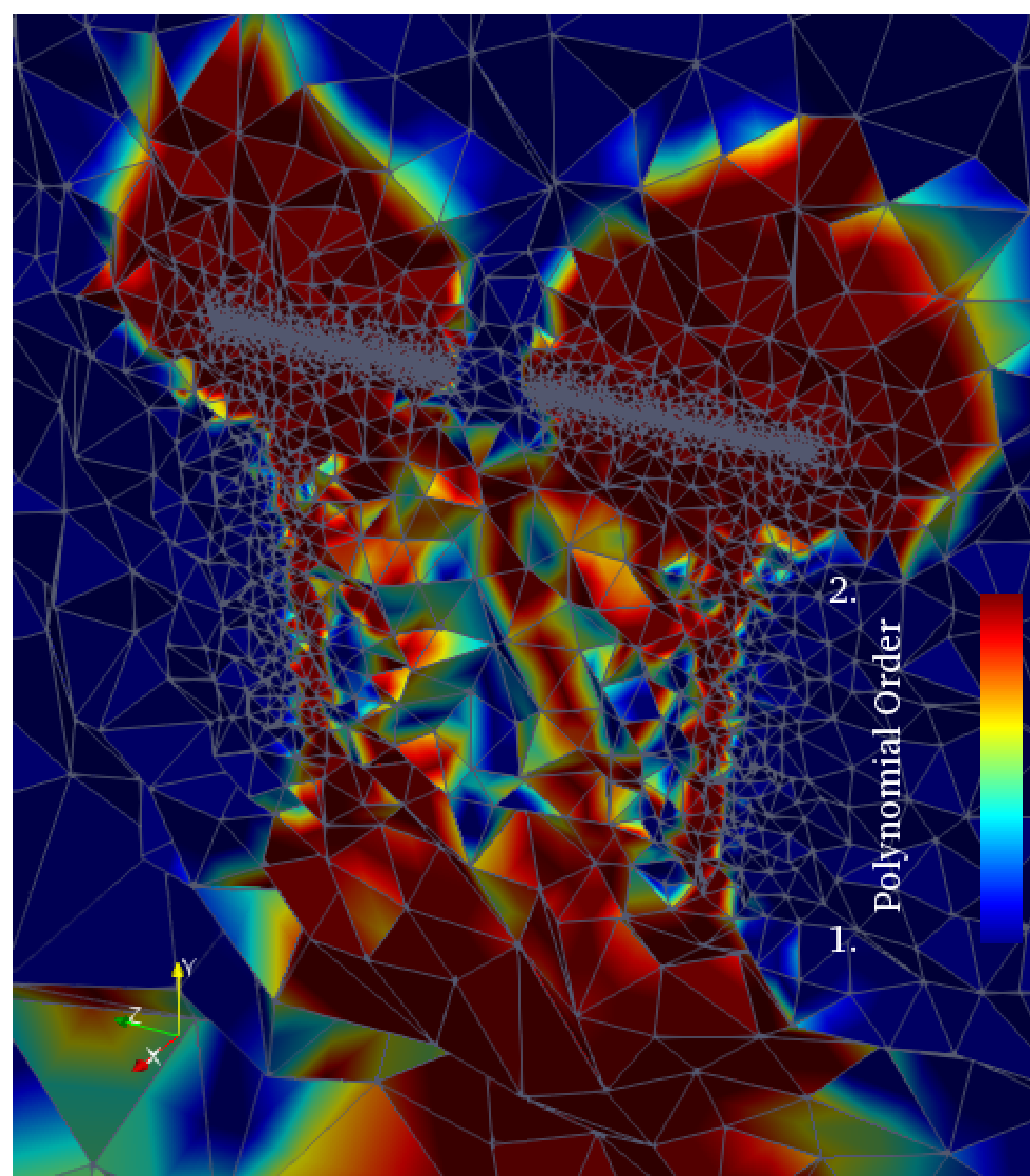
BEYOND EXPLICIT & IMPLICIT: the exponential time discretizations for N-S equations.

**Advantages:** No CFL restriction; **10–100X** faster than explicit TVD-RK3 and implicit BDF2; Low absolute temporal error.

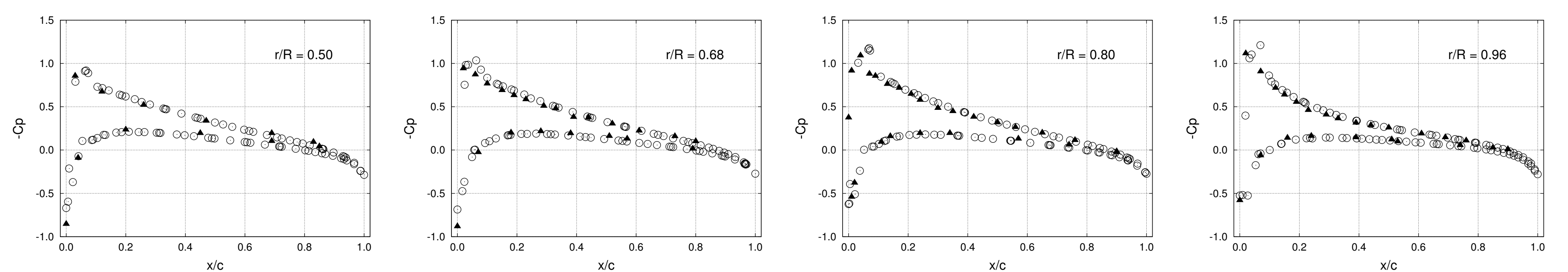
### References:

1. S.-J. Li. arXiv:1807.01151
2. S.-J. Li et al. AIAA-2019-0907
3. S.-J. Li et al. JCP, 365(2018) 206-225
4. S.-J. Li et al. AIAA-2018-0369
5. S.-J. Li et al. AIAA-2017-0753

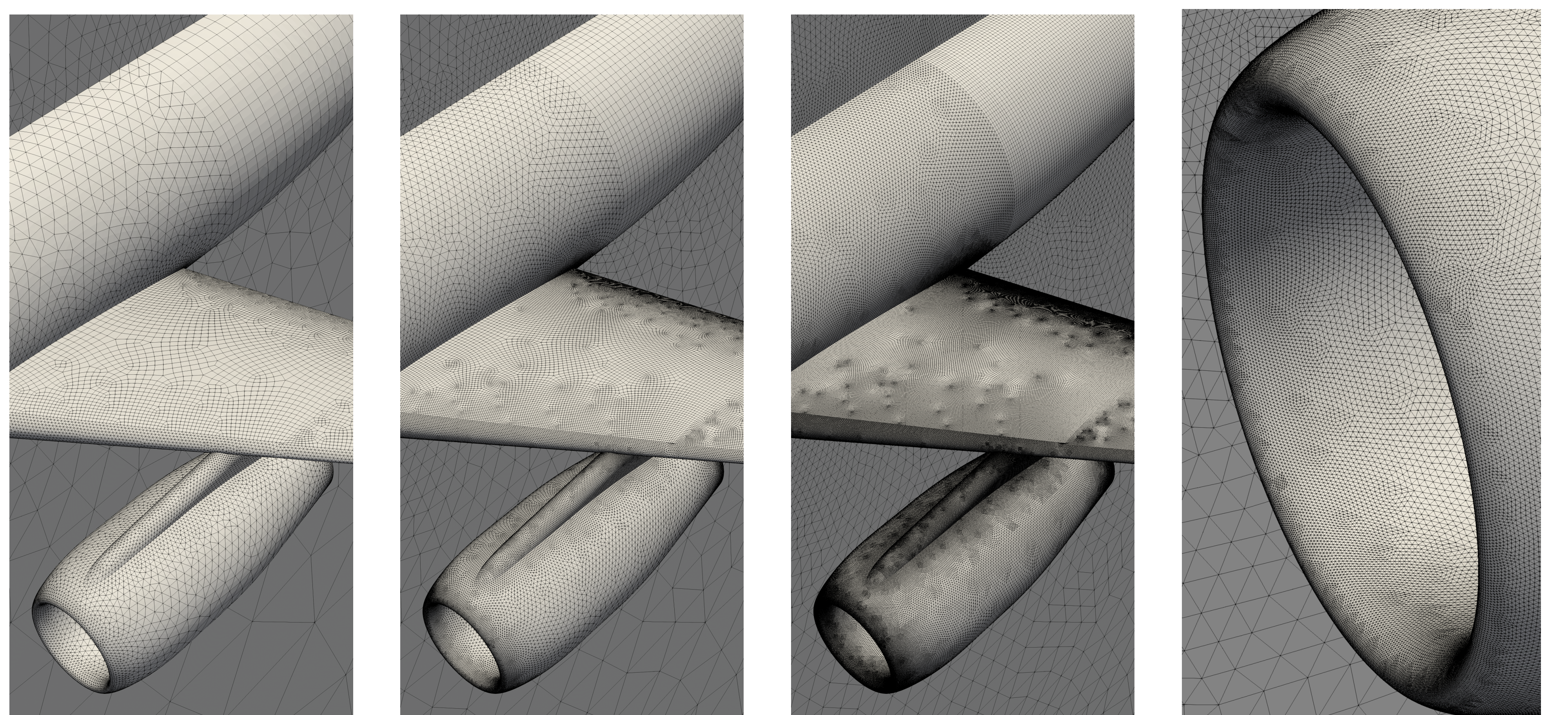
## ADAPTIVE-ORDER SIMULATIONS OF ROTOR FLOWS



*HA3D* solves tough complex 3-D fluid problems with the state-of-art methods in a fully adaptive, parallel framework. The results are in excellent agreement with experimental data.



## THE 0.9 Billion HYBRID, CURVED GRID GENERATION



Shu-Jie Li, *Mesh curving and refinement based on cubic Bézier surface for high-order discontinuous Galerkin methods*, **Computational Mathematics and Mathematical Physics** (2019 in press, a RAS journal).