Quantum criticality with two length scales

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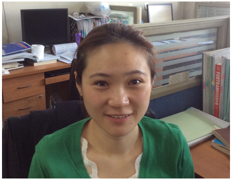
Collaborators

- Hui Shao (邵慧), Beijing Normal University (current: CSRC and BU)



- Anders W. Sandvik, Boston University

References:

1. Science: DOI: 10.1126/science.aad5007

2. PRB 91, 094426 (2015).

outlines

Background

- Deconfined quantum criticality
- introduction to finite-size scaling
- J-Q model and scaling violation

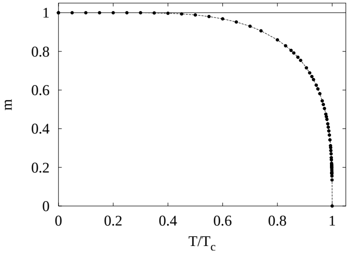
QMC study of deconfined criticality

- Quantum Monte Carlo methods
- spinon deconfinement and critical exponents
- VBS domain wall thickness
- critical scaling with two lengths and phenomenological explanation
- Anomalous critical scaling at Finite temperature

Conclusions

Thermal phase transitions

- At critical point, divergent length scale leads to singularity, which is the result of thermal fluctuations
- Quantum mechanics is largely irrelevant



3D Ising FM-Paramagnetic transition (MC simulation)

- The coarse grained continuum field description: Landau-Ginzburg-Wilson Hamiltonian

$$H(\mathbf{\Phi}) = \int \mathrm{d}V((\nabla\mathbf{\Phi})^2 + s\mathbf{\Phi}^2 + u(\mathbf{\Phi}^2)^2); \quad \mathcal{Z} = \int \mathcal{D}\mathbf{\Phi} \ \ \mathrm{e}^{-H(\mathbf{\Phi})}$$

  where $\mathbf{\Phi}$ is the order parameter, s is a function of T.
- Meanfield: $\mathbf{\Phi}^2 = -s/2u$ for $T < T_c (s \sim s'(T - T_c))$.
- well understood within Wilson's RG framework;
  - longrange order $\langle \mathbf{\Phi} \rangle \neq 0$: spontaneous symmetry breaking
  - universality class: symmetry and dimensions

Quantum phase transitions

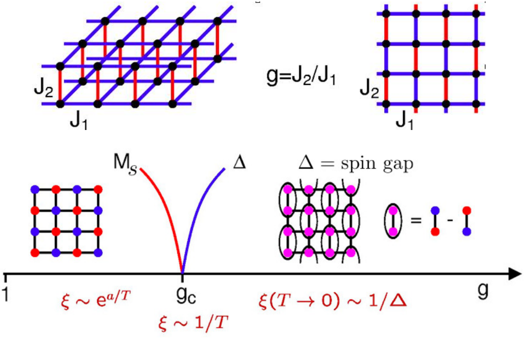
- happens at zero temperature, when adapt g in $H = H_0 + gH_I$; $[H_0, H_I] \neq 0$, continueous transition
- at g_c, the correlation length diverges, due to quantum fluctuations
- path integral maps D-dim quantum systems onto classical field theories in $D+1$-dim

$$\mathcal{S}(\mathbf{\Phi}) = \int \mathrm{d}V\mathrm{d}\tau((\partial_\tau\mathbf{\Phi})^2 + v^2(\nabla_x\mathbf{\Phi})^2 + s\mathbf{\Phi}^2 + u(\mathbf{\Phi}^2)^2)$$

$$Z = \int \mathcal{D}\mathbf{\Phi} \;\; \mathrm{e}^{-\mathcal{S}(\mathbf{\Phi})}$$

- many of these transitions can be understood in the conventional Landau-Ginzburg-Wilson framework

- for example: AF Néel-Paramagnetic transition
  H_0 is AF Heisenberg Hamiltonian, $g = J_2/J_1$



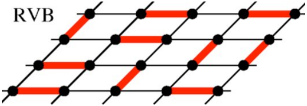
- 3D classical Heisenberg universality class: confirmed by QMC
- Experimental realized

However, many strongly-correlated quantum materials seem to defy such a description and call for new ideas

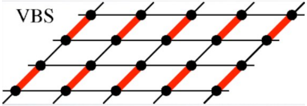
for example, continuous transition from Néel to VBS state

Non-trivial non-magnetic ground state

- Valence bond  $= (\uparrow_i\downarrow_j - \downarrow_i\uparrow_j)/\sqrt{2}$



- resonating valence-bond (RVB) spin liquid
  exotic state without any long-range order

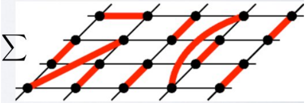


- valence-bond solid (VBS)
  breaking the translation and rotation symmetry of the lattice

VBS order parameter (D_x, D_y)

$$\boxed{D_x = \frac{1}{N}\sum_{i=1}^{N}(-1)^{x_i}\mathbf{S}_i\cdot\mathbf{S}_{i+\hat{x}},\ D_y = \frac{1}{N}\sum_{i=1}^{N}(-1)^{y_i}\mathbf{S}_i\cdot\mathbf{S}_{i+\hat{y}}}$$

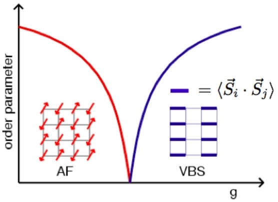
- valence-bond state: the overcomplete basis

$\sum$ 

$$\boxed{\langle \mathbf{m}_s\rangle = \langle \frac{1}{N}\sum_i \mathbf{S}_i(-1)^{x_i+y_i}\rangle = 0}$$

Deconfined quantum criticality

describe the direct continuous transition from Néel to VBS in 2D Read and Sachdev, 1989; Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)



- the direct continous transition violates the "Landau rule":

  Néel-param should be in 3D O(3) universality class;

  but transition away from VBS sould be in 3D O(2) (Z_4 anisotropy is dangerously irrelevant, Léonard and Delamotte, PRL 2015) universality class.

  either first order or a phase in between

- reason: Berry phase related interference effect in path integral, complex statistical weight in the field theories, NOT like classical statistical systems

New physics is called

Deconfined quantum criticality

Field-theory description with spinor field $\mathbf{z}$

- Order parameters of the Néel state and the VBS state are NOT the fundamental objects, they are composites of fractional quasiparticles carrying $S = 1/2$

$$\mathbf{\Phi} = z^*_\alpha \boldsymbol{\sigma}_{\alpha\beta} z_\beta$$

$\mathbf{z}$: spinor field (2-component complex vector); $\boldsymbol{\sigma}$: Pauli

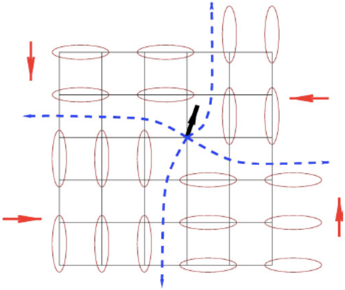
$$\mathcal{S}_z = \int \mathrm{d}r^2 \mathrm{d}\tau [|(\partial_\mu - iA_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2) + \kappa(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2]$$

$\mathbf{A}$ is a $U(1)$ symmetric gauge field; related to the VBS order parameter (D_x, D_y)
Non-compact CP^1 action

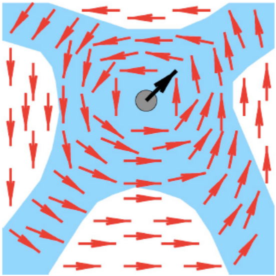
Physical picture

Levin and Senthil, PRB 70, 2004

At the core of the Z_4 vortex, there is a spinon



Blue-shaded regions are domain walls (Not a line). The thickness ξ_{DW} diverges faster than the correlation length ξ: two length scales , emergent $U(1)$ symmetry



- Spinons bind together in the VBS state (confinement) and condensate the Néel state, deconfine at the critical point leading to a continuous phase transition

- Only SU(N) generalization can be sloved when $N \to \infty$, nonperturbative numerical simulations are required to study small N

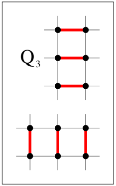
- The most natural physical realization of the Néel-VBS transition for SU(2) spins is in **frustrated quantum magnets**

  however, notoriously difficult to study numerically: sign problem in QMC

Designer Hamiltonian: J-Q model

Sandvik designs the J-Q model

$$H = -J \sum_{\langle ij \rangle} P_{ij} - Q \sum_{\langle ijklmn \rangle} P_{ij} P_{kl} P_{mn}, \quad P_{ij} = (\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j)$$



Lattice symmetries are kept ($J - Q_2$ version similar)

- large Q, columnar VBS

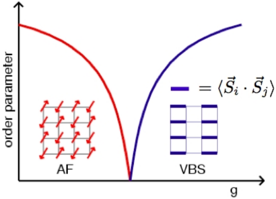
  VBS order parameter

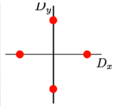
  $$D_x = \frac{1}{N} \sum_{i=1}^{N} (-1)^{x_i} \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}},$$
  $$D_y = \frac{1}{N} \sum_{i=1}^{N} (-1)^{y_i} \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}}$$

- small Q, Néel

  Néel order parameter

  $$\mathbf{m}_s = \frac{1}{N} \sum_i \mathbf{S}_i (-1)^{x_i + y_i}$$





- No sign problem for QMC simulations,
- ideal for QMC study of the DQC physics

Sandvik, PRL 98, 227202(2007)

Finite-size scaling

- Correlation length divergent for $T \to T_c$: $\xi \propto |\delta|^{-\nu}, \delta = T - T_c$
- Other singular quantity: $A(T, L \to \infty) \propto |\delta|^{\kappa} \propto \xi^{-\kappa/\nu}$
- For L-dependence at T_c just let $\xi \to L$: $A(T \approx T_c, L) \propto L^{-k/\nu}$
- Close to critical point: $A(T, L) = L^{-\kappa/\nu} g(L/\xi) = L^{-\kappa/\nu} f(\delta L^{1/\nu})$

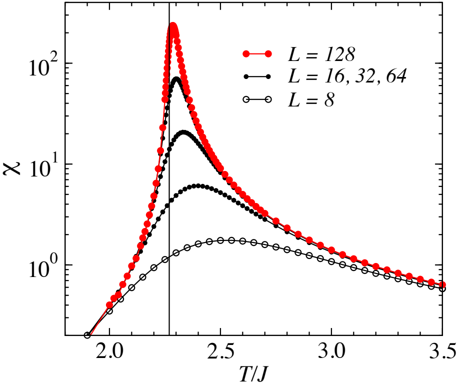
For example

$$\chi(T, L \to \infty) \propto \delta^{-\gamma}$$

data collapse

$$\chi(T, L) L^{-\gamma/\nu} = f(\delta L^{1/\nu})$$

2D Ising model $\gamma = 7/4, \nu = 1$
$T_c = 2/\ln(1+\sqrt{2}) \sim 2.2692$
When these are not known, treat as fitting parameters



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For example

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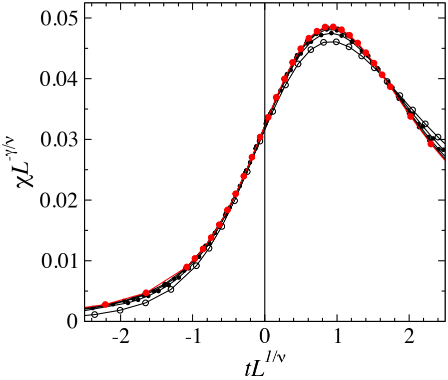


systematic critical-point analysis

- consider a quantity with $\kappa = 0$, or, with known κ/ν

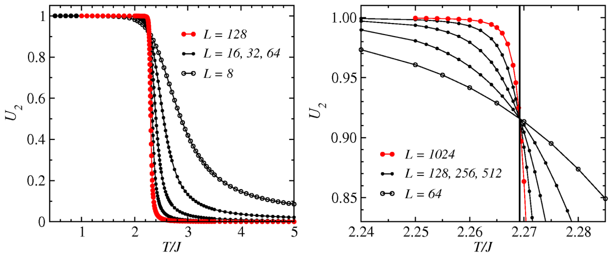
$$AL^{\kappa/\nu} = f(\delta L^{1/\nu}, u_1 L^{-\omega_1}, u_2 L^{-\omega_2}, \ldots)$$

corrections to scaling are included (RG theory); u_i are irrelevant fields

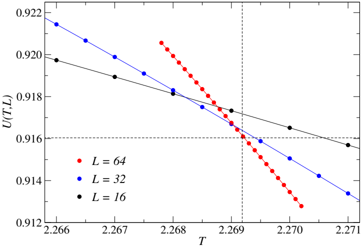
- (almost) size-independent at T_c leads to crosssings at T_c

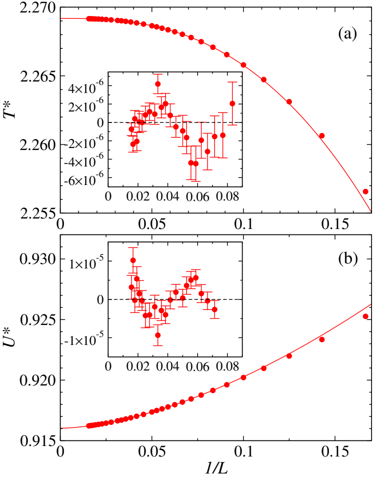
Binder cumulant $U = \frac{1}{2}(3 - \frac{\langle m^4 \rangle}{\langle m^2 \rangle^2})$, dimensionless $\kappa = 0$

2D Ising model; MC results



$$Q(T,L) = f(\delta L^{1/\nu}, uL^{-\omega})$$





Drift in $(L, 2L)$ crossing points

- scaling corrections in crossings

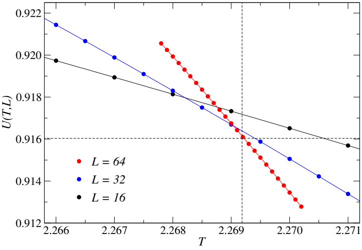
$$T^* = T_c + aL^{-(1/\nu+\omega)}$$

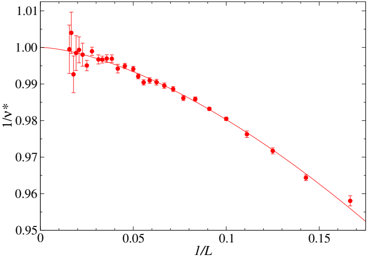
$$U* = U_c + bL^{-\omega}$$

ω: unkown correction to scaling, free exponent in fits

- correlation-length exponent ν
  can be extracted from the slope of U: $s(T,L) = \frac{\mathrm{d}U(T,L)}{\mathrm{d}T}$

$$\ln\left(\frac{s(T^*,2L)}{s(T^*,L)}\right)/\ln 2 = \frac{1}{\nu} + aL^{-\omega} + \cdots$$





numerical study of the J-Q model

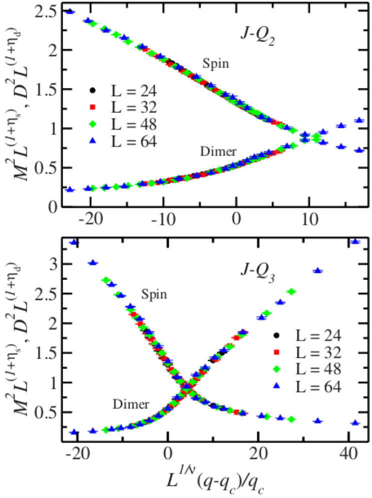
FSS of squared order parameter(A)

$$A(q,L) = L^{-(1+\eta)} f[\delta L^{1/\nu}], \qquad \delta = q - q_c, (q = Q/(J+Q))$$

Data "collapse":
M^2 and D^2 simutaneously $\rightarrow$ single continous transition!

- J-Q_2 model; $q_c = 0.961(1)$
  $\eta_s = 0.35(2); \eta_d = 0.20(2)$;
  $\nu = 0.67(1)$
- J-Q_3 model; $q_c = 0.600(3)$
  $\eta_s = 0.33(2); \eta_d = 0.20(2)$;
  $\nu = 0.69(2)$ Lou,Sandvik and Kawashima, PRB 2009
- Comparable results for honeycomb J-Q model

Alet and Damle, PRB 2013 Kaul et al., PRL 2014



scaling violation

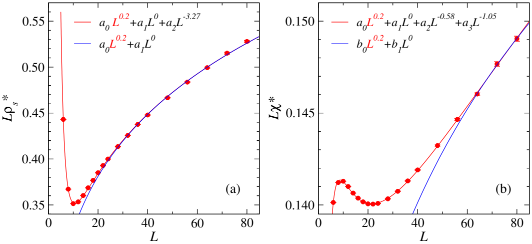
Spin stiffness $\rho_s \propto \delta^{\nu(d+z-2)}$, susceptibility $\chi \propto \delta^{(d-z)\nu}$

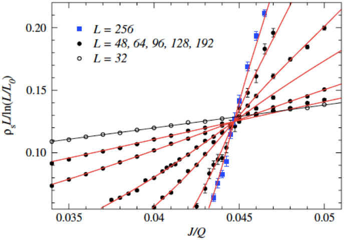
Conventional FSS

$$\boxed{\rho_s(\delta, L) = L^{-\nu(d+z-2)/\nu} f(\delta L^{1/\nu}), \qquad \chi(\delta, L) = L^{-\nu(d-z)/\nu} f(\delta L^{1/\nu})}$$

At critical point: $\rho_s \propto L^{-(d+z-2)} = L^{-z}, \quad \chi \propto L^{-(d-z)}$

$z = 1$ for J-Q model, $\rho_s L$ and χL should be constants at q_c





- $z \neq 1$ **does not work**
- **large scaling corrections?** Sandvik PRL 2010, Bartosch PRB 2013
- **weak first-order transition?** Chen et al PRL 2013

The enigmatic current state is well summed up in

Nahum et al arXiv: 1506.06798

In this talk, we will try to resolve this puzzle

- **study the deconfinement of spions**
- **VBS domain wall thickness**
- **introduce scaling form with two-length scales and give phenomenological explanation**
- **anomalous critical scaling at finite temperature**

Quantum Monte Carlo method

General idea of QMC :

- rewrite a quantum-mechanical trace or expectation value into a classical form

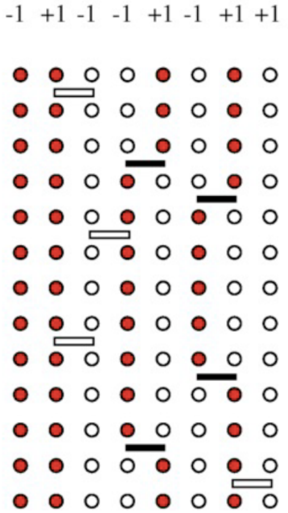
$$\langle A \rangle = \frac{\mathrm{Tr}\{A\mathrm{e}^{-\beta H}\}}{\mathrm{Tr}\ \mathrm{e}^{-\beta H}} \ \text{ or } \ \frac{\langle \Psi | A | \Psi \rangle}{\langle \Psi | \Psi \rangle} \rightarrow \frac{\sum_c A_c W_c}{\sum_c W_c}$$

W_c is the weight of a configuration, A_c is the estimator of A.

- There are many different ways of doing it:
  Worldline (worm), SSE, Fermion determinant, $\cdots$

SSE Quantum Monte Carlo method

- A SSE configuration: spin state + operator string



- diagonal and loop updates
- observables and estimators
  - energy estimator : number of operators, $H_c = -n/\beta$
  - spin stiffness estimator : winding number fluctuations

$$\rho_s = \frac{\langle W_\alpha^2 \rangle}{L^{d-2}\beta}$$

  - staggered magnetization: $m_{sz} = \sum_i (-1)^{i_x+i_y} s_{iz}/N$

Projector Quantum Monte Carlo method

For ground state calculations

Apply the imaginary time evolution operator to an initial state

$$U(\tau \to \infty)|\Psi_0\rangle \to |0\rangle$$

where $U(\tau) = (-H)^\tau$ or $U(\tau) = \exp(-H\tau)$

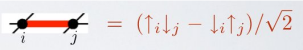
$$\boxed{\langle A \rangle = \frac{\langle \Psi_0 | U(\tau) A U(\tau) | \Psi_0 \rangle}{\langle \Psi_0 | U(\tau) U(\tau) | \Psi_0 \rangle} \to \frac{\sum_c A_c W_c}{\sum_c W_c}}$$

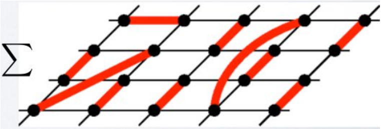
A_c is the estimator of A.

Projector Quantum Monte Carlo method

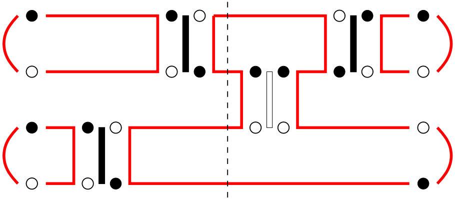
- using VB basis (in the singlet sector)

$$|\Psi\rangle = \sum_{v} f_v |v\rangle, \qquad |v\rangle = |(a_1, b_1)\cdots(a_{N/2}, b_{N/2})\rangle$$



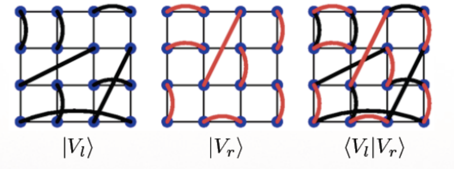


- take $U(\tau) = \exp(-\tau H)$, SSE representation $\rightarrow Z = \sum_c W_c$
- loop update algorithm are used



Expectation values

- energy estimator: $H_c = -n/2\tau$
- correlation functions computed using transition graphs



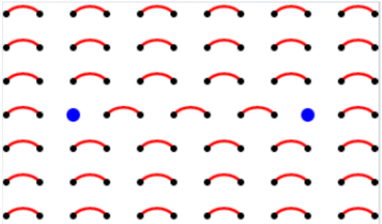
$$\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle = \left\{ \begin{array}{ll} 0, & (i)_L(j)_L \\ \frac{3}{4}\phi_{ij}, & (i,j)_L, \end{array} \right.$$

$\phi_{ij} = \pm 1$, i,j on the same/different sublattice

spinons and Deconfinement of spinons

- The excitations of VBS carry $S = 1$
  - bound spinon pair
  - confining string
- The confining string weakens as q_c is approached
  - deconfinement

    The distance Λ diverages, $\propto \xi_{DW}$(?)



QMC simulations can be carried out in the $S = 1$ space

Extend valence-bond basis to total spin $S > 0$ states

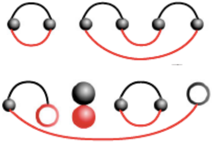
Tang and Sandvik PRL 2011, Banerjee and Damle JSTAT 2010

Consider $S_z = S$

- for even N spins: $N/2 - S$ bonds, $2S$ upaired "up" spins

$S = 0$

$S = 1$



$\langle V_\beta | V_\alpha \rangle$, 2 loops

$\langle V_\beta(j,l) | V_\alpha(i,k) \rangle$,
1 loop, 2 strings

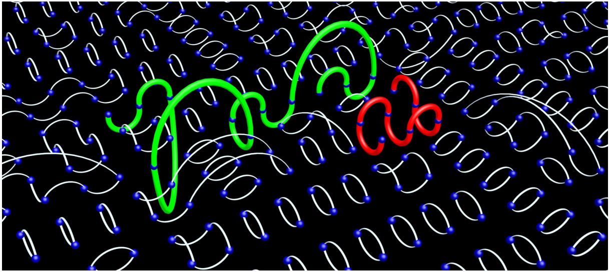
- transition grap has $2S$ open strings

study spinon bound states and unbinding

The two-spinon distance in the J-Q_2 model

A QMC transition graph representing $\langle \psi_L | \psi_R \rangle$ of $S = 1$ states

- two strings (spinons) in a background of loops formed by valence bonds.
- two strings represent two spinons in bound state



J-Q model in deep VBS phase

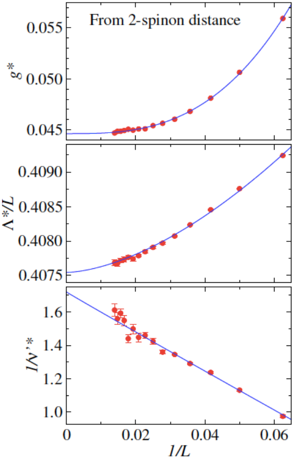
animation at VBS

J-Q model at the critical point

animation at q_c

The two-spinon distance in the J-Q_2 model

Define the size of spinon bound state Λ as root-mean-square string distance



Crossing-point analysis of Λ/L

- We find that Λ/L at q_c is dimensionless, like the Binder ratio R_1 of Néel order parameter.
- The crossing points of $(L, 2L)$ converge monotonicly

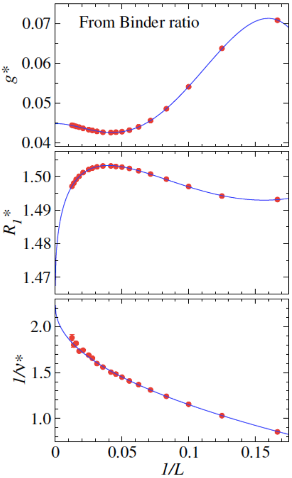
$$g^* - q_c \propto L^{-(1/\nu' + \omega)}, \quad \Lambda^*(L)/L - R \propto L^{-\omega}$$

$1/\nu'$ can be extracted from slopes at the crossing point

- we find $q_c = 0.04463(4), \nu' = 0.58(2)$

Transition is associated with spinon deconfinement

The Binder ratio in the J-Q_2 model



Similar crossing-point analysis of the Binder ratio

- From Binder ratio, we have $\nu = 0.446$ which controls the correlation.
- what is ν'?
  - DQC theory: two diverging length scales

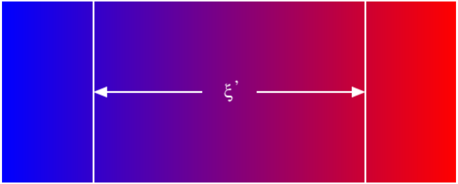
$$\xi \propto (q - q_c)^{-\nu}, \qquad \xi_{DW} \propto (q - q_c)^{-\nu'}, \nu' > \nu$$

  - $\nu/\nu' = 0.77(3)$ agrees with the result obtained from the VBS domain-Wall energy calculations suggesting ν' is the domain wall thickness exponent

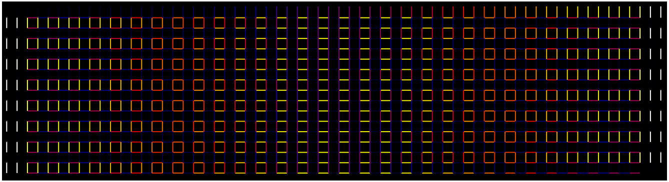
fundamental lenght scale: domain wall thickness

In some classical systems (clock models, XY model with symmetry breaking field) the thickness of a domain wall is larger than the correlation length

$$\xi \sim \delta^{-\nu}, \qquad \xi_{DW} \sim \delta^{-\nu'}, \qquad \nu' > \nu$$

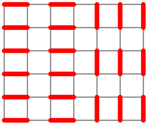


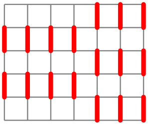
VBS domain wall behaves similarly



VBS domain-wall scaling in the critical J-Q model

$\phi = \pi/2$ $\phi = \pi$



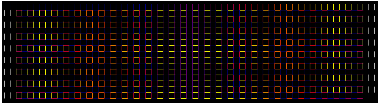


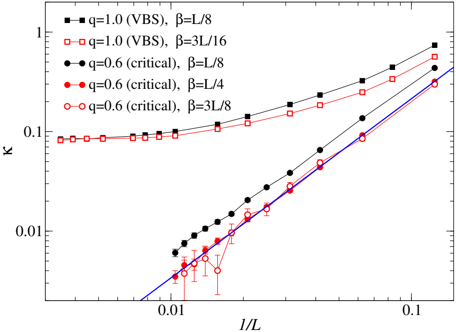
two kinds of VBS domain walls are imposed in open-boundary systems

π wall splits into two $\pi/2$ walls

$$\delta F = F_{wall} - F_{uniform}$$

$$\kappa = \delta F / L^{d+z-1}$$





At deconfined critical point q_c, $\kappa \propto L^{-b}, b \approx 1.80(1)$

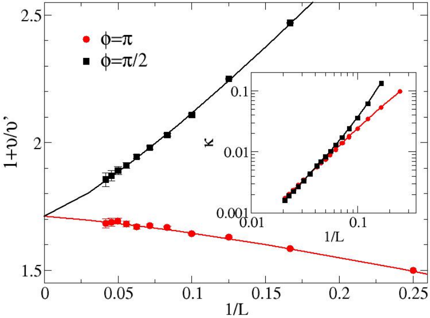
Shao, Guo and Sandvik, PRB 2015

At deconfined critical point

- domain-wall energy can be expressed as $\kappa = \rho_s/\Lambda$
  ρ_s is a stiffness: energy cost of a twist of the VB order
  Λ is the width of the region over which the twist distributes.

- According to DQC theory,
  $\rho_s \propto 1/\xi$, $\Lambda \propto \xi_{DW}$
  $\kappa \propto \frac{1}{\xi\xi_{DW}} \propto (q-q_c)^{\nu+\nu'}$
- translate to finite size at q_c:
  $\xi_{DW} = L, \xi = \xi_{DW}^{\nu/\nu'}$

$$\kappa(q_c) \propto L^{-(1+\nu/\nu')}$$



we have $b = 1+\nu/\nu'$, and $\nu/\nu' = 0.80(1)$

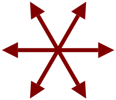
- The only other estimate from analysis of the emergent U(1) symmetry: $\nu/\nu' = 0.83((4)$, J. Lou et al PRB 80, 180414(R)(2009)

Domain wall scaling in classical model

3D q-state clock model$(q > 3)$: basic example of dangerously irrelevant perturbation

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

- θ restriction:



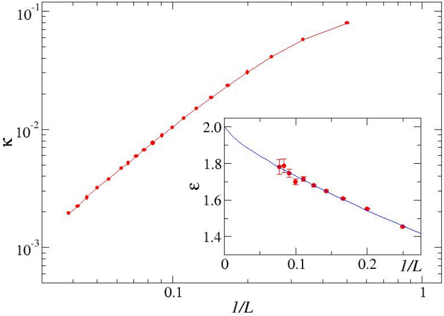
The prediction for the domain wall energy in $L \to \infty$

$$\kappa \sim \frac{1}{\xi \xi_{DW}}$$

But, finite-size scaling at T_c shows

$$\kappa \sim L^{-2} \neq L^{-(1+\nu/\nu')}$$

$\xi \sim \xi_{DW}^{\nu/\nu'}$, $\nu/\nu' \approx 0.47$, ν' is universal Léonard and Delamotte, PRL 2015



The dangerously irrelevant perturbation in the J-Q model is more serious

Quantum criticality with two lengths

Two divergent lengths tuned by one parameter:

$$\xi \propto \delta^{-\nu}, \quad \xi' \propto \delta^{-\nu'}$$

A quantity $A \propto \delta^{\kappa}$, finte-size scaling of A

- Conventional scenario

$$A(\delta, L) = L^{-\kappa/\nu} f(\delta L^{1/\nu}, \delta L^{1/\nu'}), \quad A(\delta = 0, L) \propto L^{-\kappa/\nu}$$

When $L \to \infty, f(\delta L^{1/\nu}, \delta L^{1/\nu'}) \to (\delta L^{1/\nu})^{\kappa}$

- We propose

$$A(\delta, L) = L^{-\kappa/\nu'} f(\delta L^{1/\nu}, \delta L^{1/\nu'}), \quad A(\delta = 0, L) \propto L^{-\kappa/\nu'}$$

When $L \to \infty, f(\delta L^{1/\nu}, \delta L^{1/\nu'}) \to (\delta L^{1/\nu'})^{\kappa}$

Example: spin stiffness $\rho_s \propto \delta^{\nu(d+z-2)}, \kappa = \nu(d+z-2)$. At q_c

$\rho_s \propto L^{-(d+z-2)}$ or $\rho_s \propto L^{-(d+z-2)\nu/\nu'}$

General scaling theory for ρ_s and χ, single length scale

Fisher et al PRB,40,546(1989)

Free energy density scales

$$f_s(\delta, L, \beta) \sim \xi^{-(d+z)} Y(\frac{\xi}{L}, \frac{\xi^z}{\beta}), \qquad \xi \sim \delta^{-\nu}$$

- $\rho_s \frac{\Delta^2 \phi}{L^2}$ is the excess energy due to a twist along apace:

$$\Delta f(\delta, L, \beta) \sim \xi^{-(d+z)} \tilde{Y}(\frac{\xi}{L}, \frac{\xi^z}{\beta}) \sim \rho_s \frac{\pi^2}{L^2}$$

  $\tilde{Y}$ has to behave like $(\xi/L)^2$, thus

$$\rho_s \sim \xi^{2-(d+z)}$$

  replacing ξ to L, we have $\rho_s \sim L^{-(d+z-2)}$
- Simmilarly, $\chi \frac{\Delta^2 \phi}{\beta^2}$ is the excess energy density needed to enforce the twist, which means $\tilde{Y} \sim \xi^{2z}/\beta^2$

$$\chi \sim \xi^{2z-(d+z)}, \qquad \chi \sim L^{-(d-z)}$$

Two correlation lengths scenario

Free energy density scales

$$f_s(\delta, L, \beta) \sim \xi^{-(d+z)} Y(\frac{\xi}{L}, \frac{\xi^z}{\beta}, \frac{\xi'}{L}, \frac{\xi'^z}{\beta})$$

the excess energy due to a twist along apace:

$$\rho_s(\frac{\Delta\phi}{L})^2 \sim \Delta f(\delta, L, \beta) \sim \xi^{-(d+z)} \tilde{Y}_s(\frac{\xi}{L}, \frac{\xi^z}{\beta}, \frac{\xi'}{L}, \frac{\xi'^z}{\beta})$$

which means

$$\tilde{Y}_s \sim (\frac{\xi}{L})^a (\frac{\xi'}{L})^{2-a}$$

The larger correlation length ξ' reaches L first, so $L \sim \xi' \sim \delta^{-\nu'}$, we have $a = 2$, and

$$\rho_s \sim \xi^{-(d+z-2)}$$

but, since $L = \xi', \xi = L^{\nu/\nu'}$,

$$\rho_s \sim L^{-(d+z-2)\nu/\nu'}$$

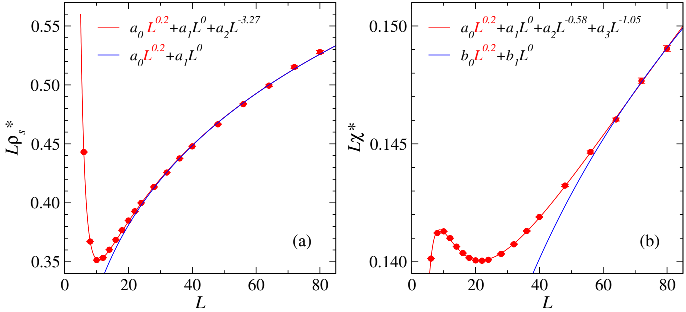
Similarly, we have

$$\chi \sim L^{-(d-z)\nu/\nu'}$$

Evidence for unconventional scaling in J-Q model

We have

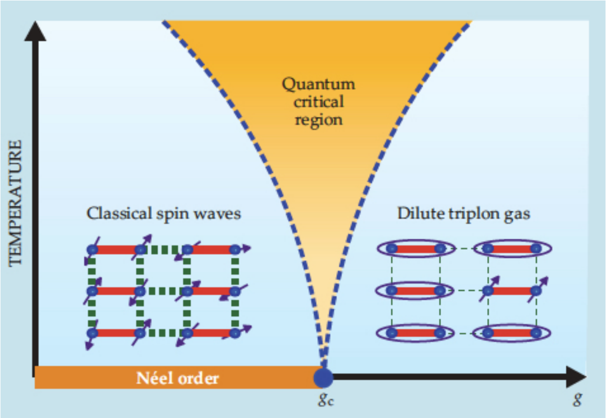
$$\rho_s \sim L^{-(z+d-2)\nu/\nu'} \sim L^{-\nu/\nu'}, \quad \text{instead of } \rho_s \sim L^{-(z+d-2)} \sim L^{-1}$$
$$\chi \sim L^{-(d-z)\nu/\nu'} \sim L^{-\nu/\nu'}, \quad \text{instead of } \chi \sim L^{-(d-z)} \sim L^{-1}$$



- this unexplains drifts in $L\rho_s$ and χL in J-Q and other models $(z = 1, d = 2)$
- **Behavior was interpreted as flow to first-order transition is due to unconventional scaling!**

Anomalous critical scaling at finite Temperature

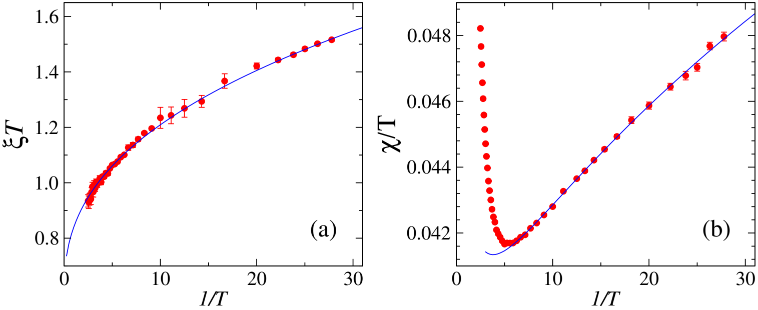
Quantum critical point at $T = 0$ governs the behavior in a $T > 0$ region which expands out from $(g_c, T = 0)$: experimentally important



Anomalous critical scaling at finite Temperature

- $\beta = 1/T$ is also a 'finite-size': $L \to \beta^{1/z}$
- convetional scaling ($z = 1$ for J-Q)
  - $\xi \sim L$ leads to $\xi_T \propto \beta^{1/z} = T^{-1}$,
  - $\chi \sim L^{-(d-z)}$ leads to $\chi_T \propto \beta^{-(d-z)/z} = T$
- new scaling with ν/ν':

$\xi_T \propto T^{-\nu'/\nu}$; $\chi \sim L^{-\nu/\nu'}$ leads to $\chi_T \propto T^{\nu/\nu'}$



Conclusions

- Two length scales observed explicitly in the J-Q model
- No sign of first-order transition in the J-Q model
- Simple two-length scaling hypothesis explains scaling violation of spin stiffness and susceptibility

- For $T > 0$ we find scaling laws from finite-size scaling forms experimentally important

Thank you !