Fundamental Problems in Quantum Non-Equilibrium Dynamics I

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CSRC Workshop on Quantum Non-Equilibrium Phenomena
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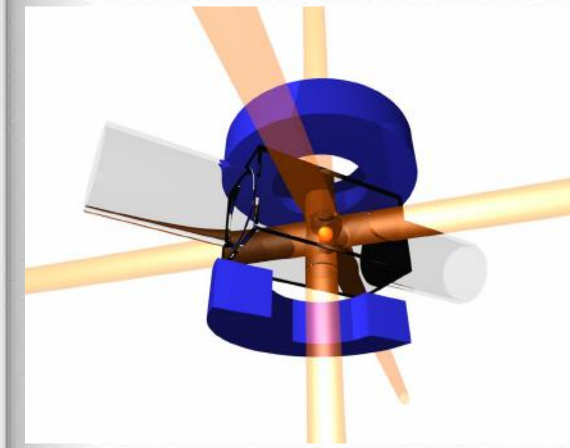
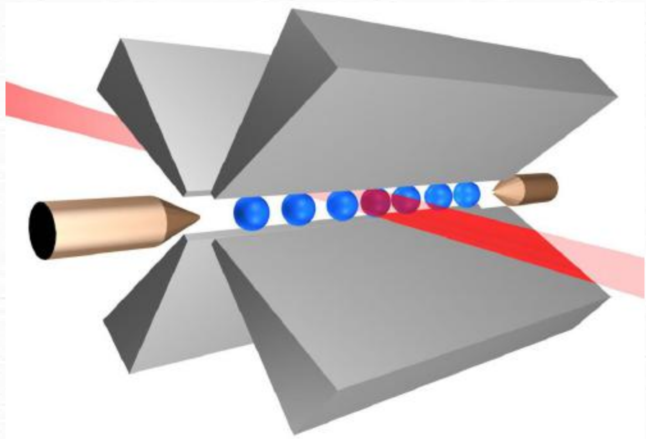
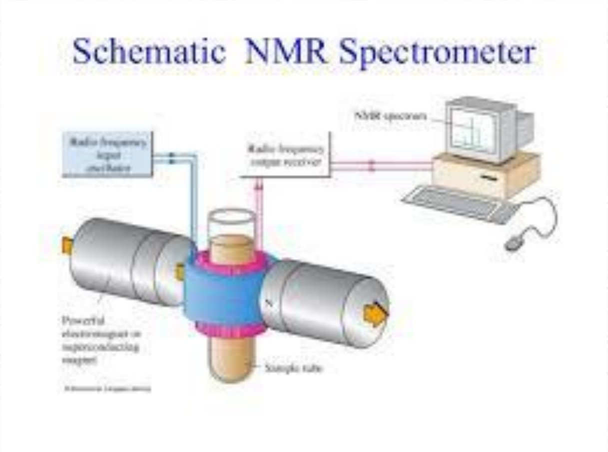
Synthetic Quantum Matter



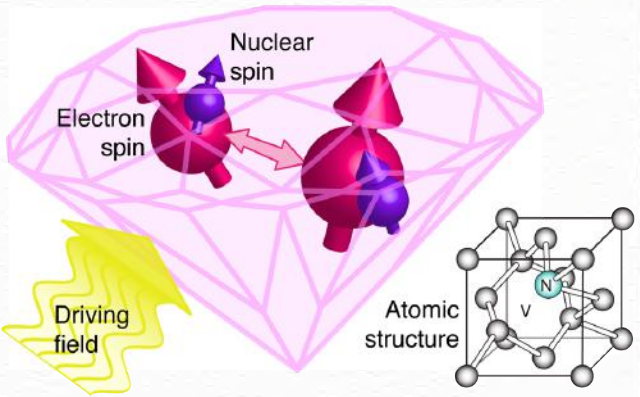
Cold Atoms



Trapped Ion

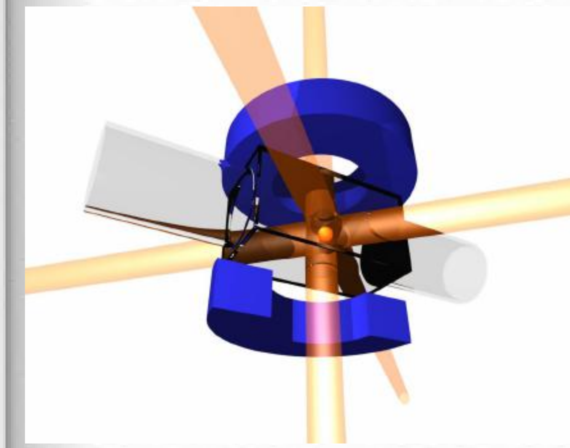


NMR

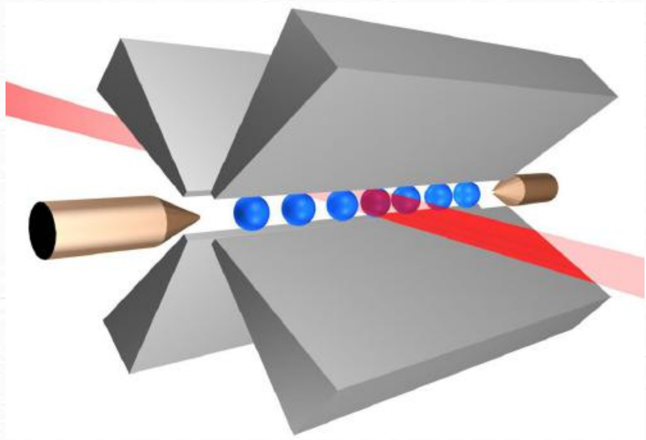


NV Center

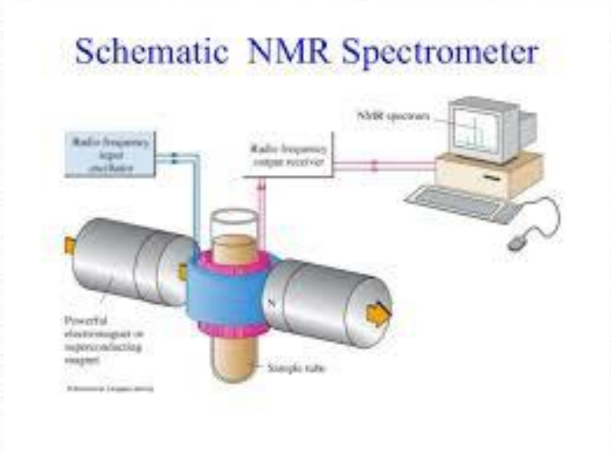
Synthetic Quantum Matter



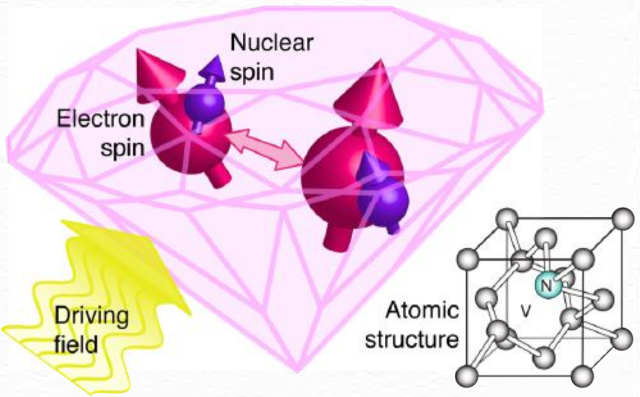
Cold Atoms



Trapped Ion



NMR



NV Center

v.s. Solid State Quantum Materials

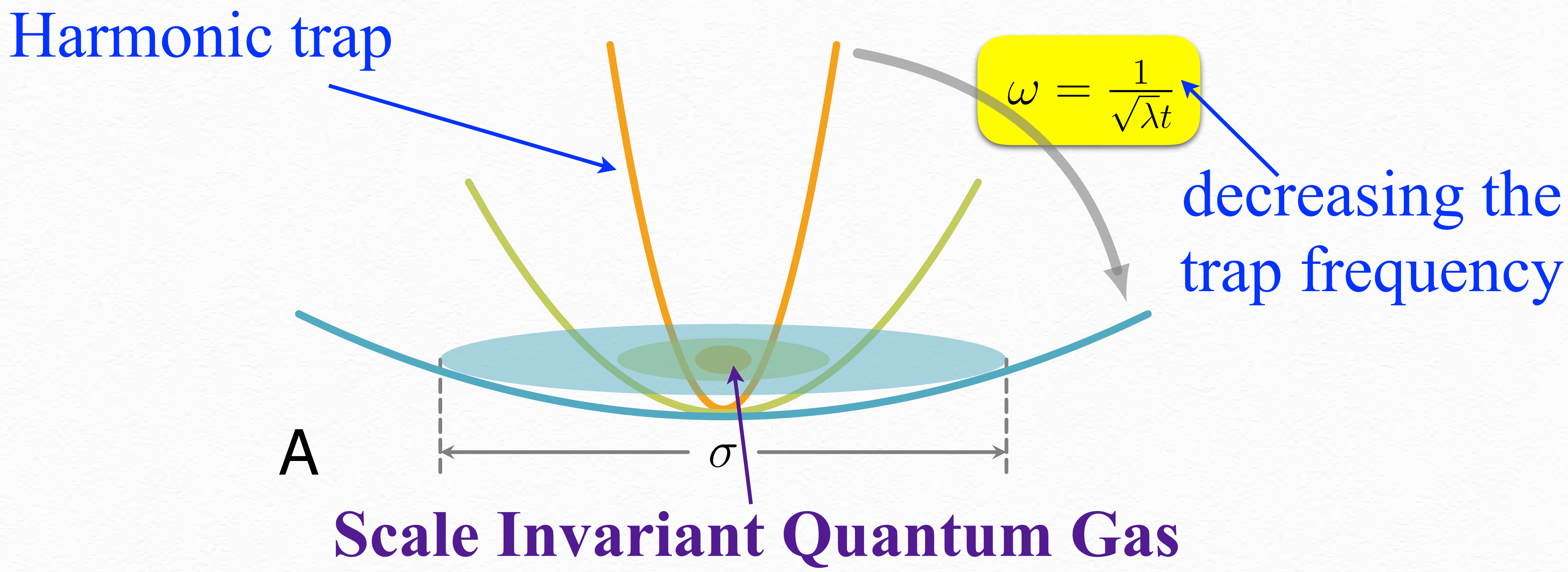
Quantum Dynamics

Non-Equilibrium Dynamics

- Simple
- Fundmental
- Universal
- Directly Relevant to Experiments
- Mathematically Solid/Rigorous

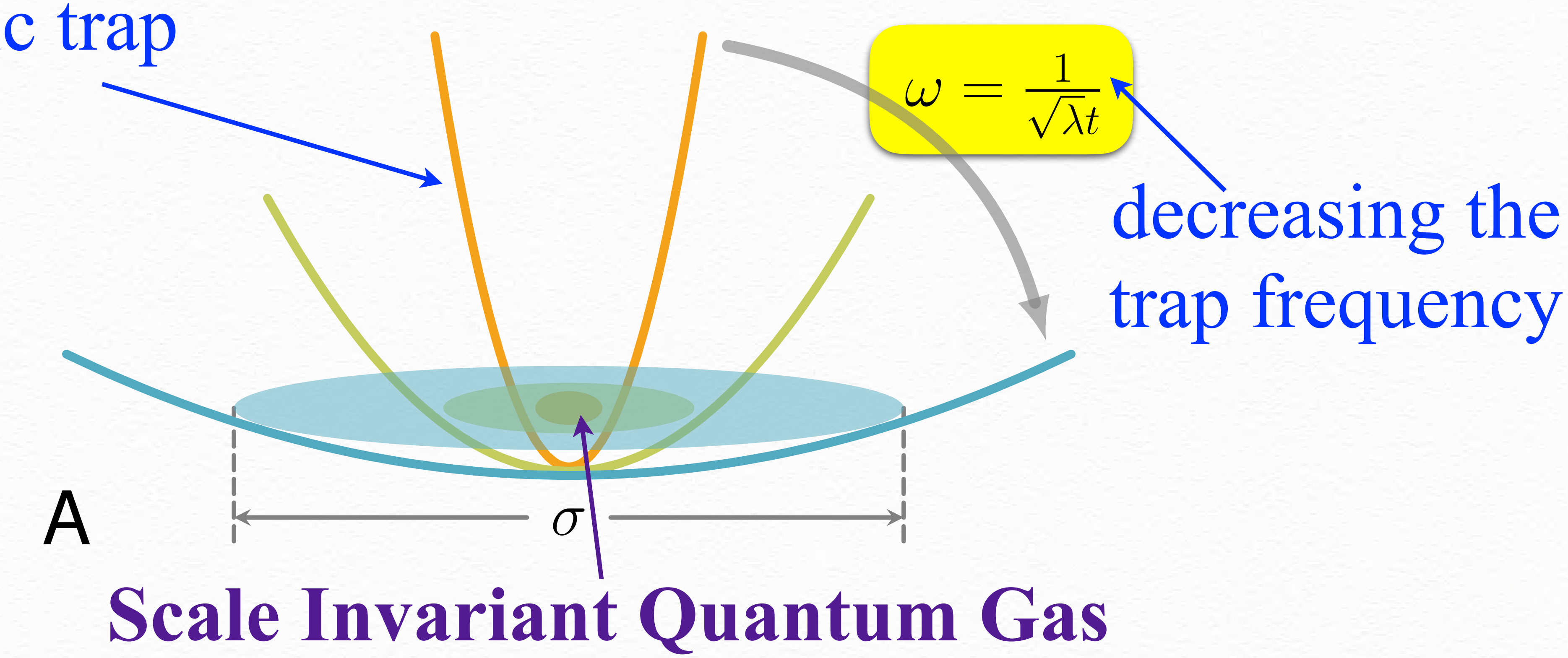
Symmetry

Expanding Harmonic Trap

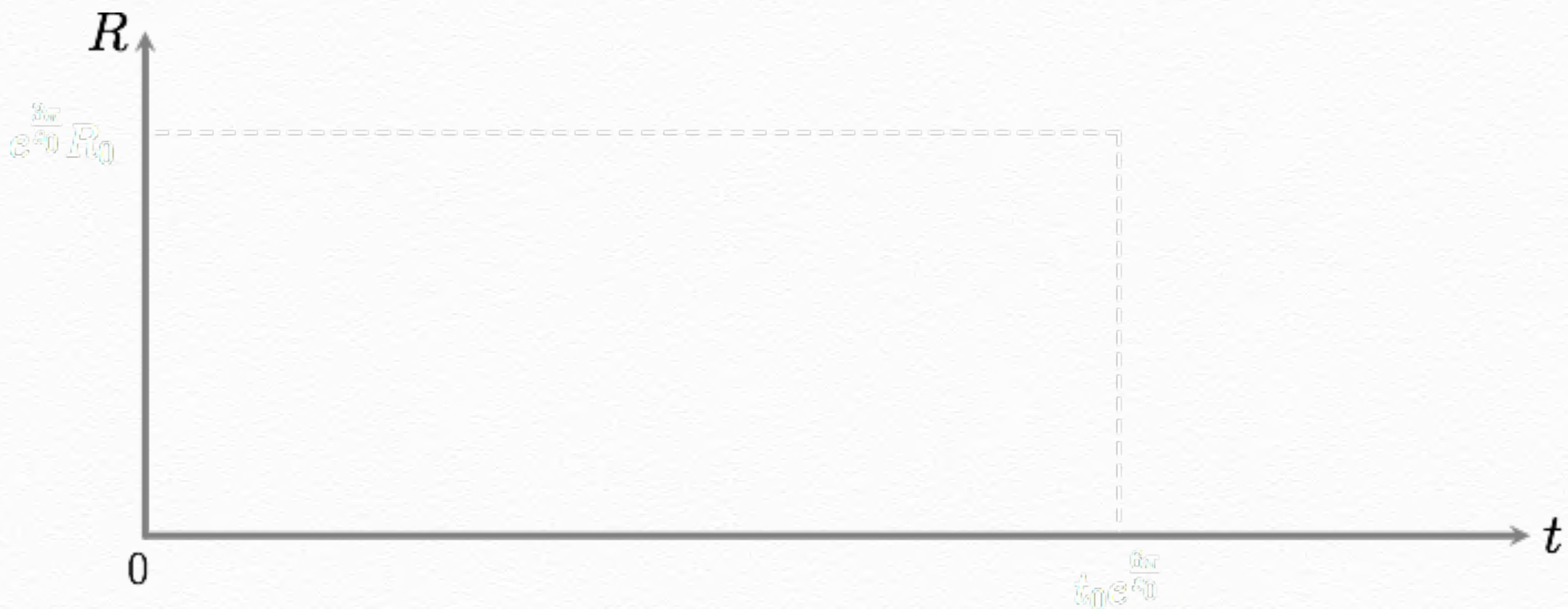


Expanding Harmonic Trap

Harmonic trap

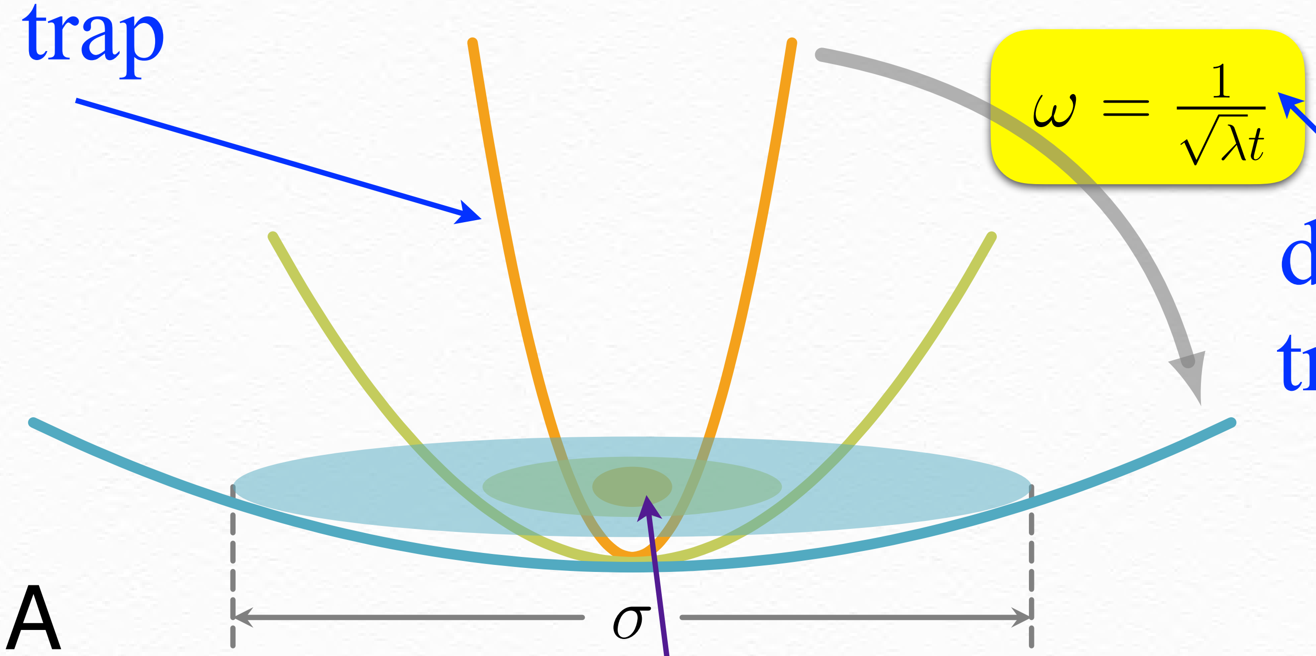


$$R = \sqrt{\langle \sum r_i^2 \rangle}$$



Expanding Harmonic Trap

Harmonic trap



decreasing the trap frequency

$$R = \sqrt{\langle \sum r_i^2 \rangle}$$

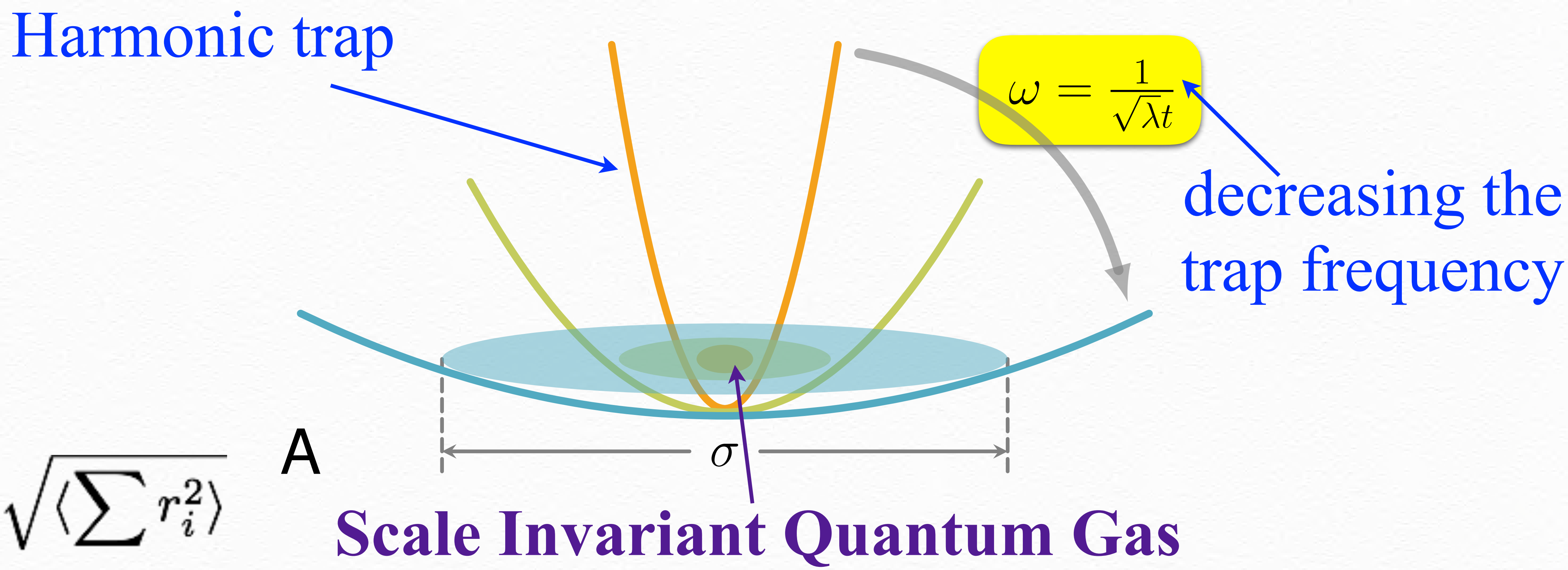
A

Scale Invariant Quantum Gas

Harmonic length: $$a = \sqrt{\frac{\hbar}{m\omega}}$$

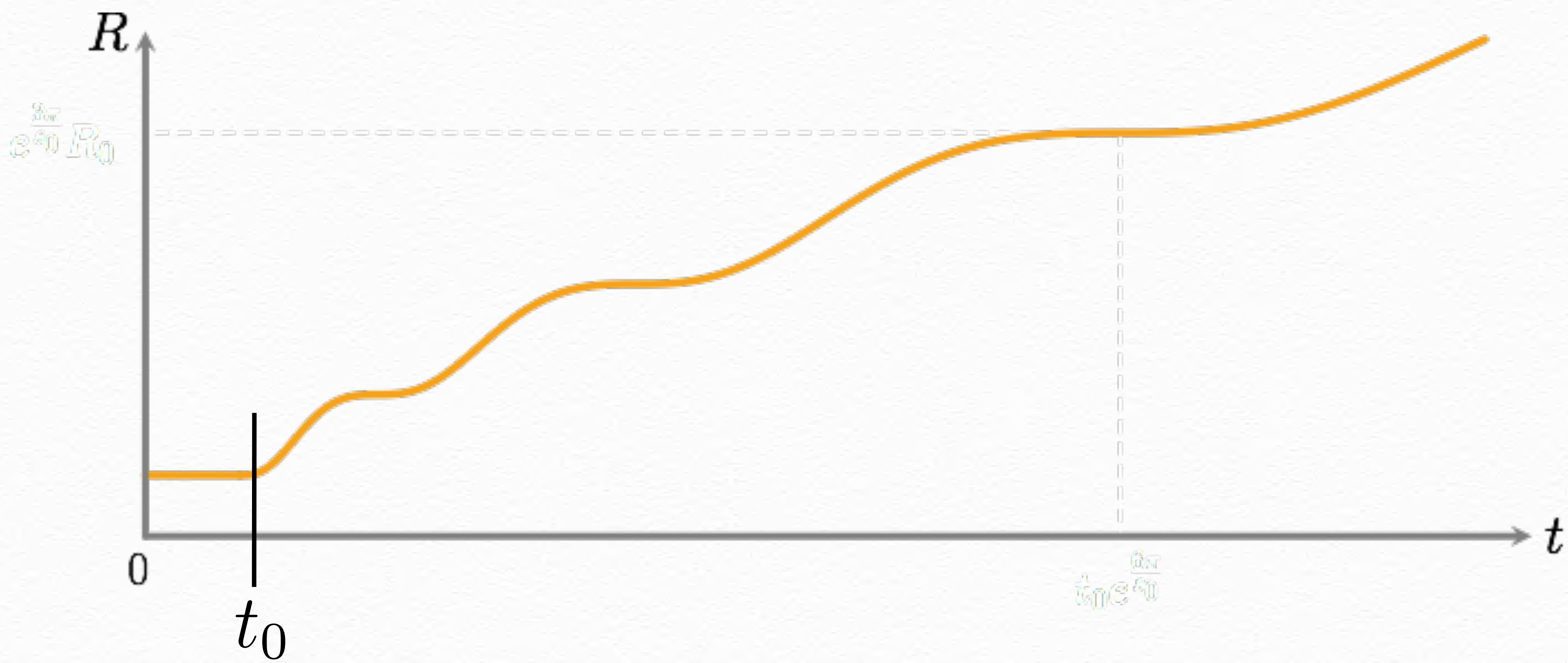
By dimension analysis: $$\mathcal{R} \sim \sqrt{t}$$

Expanding Harmonic Trap

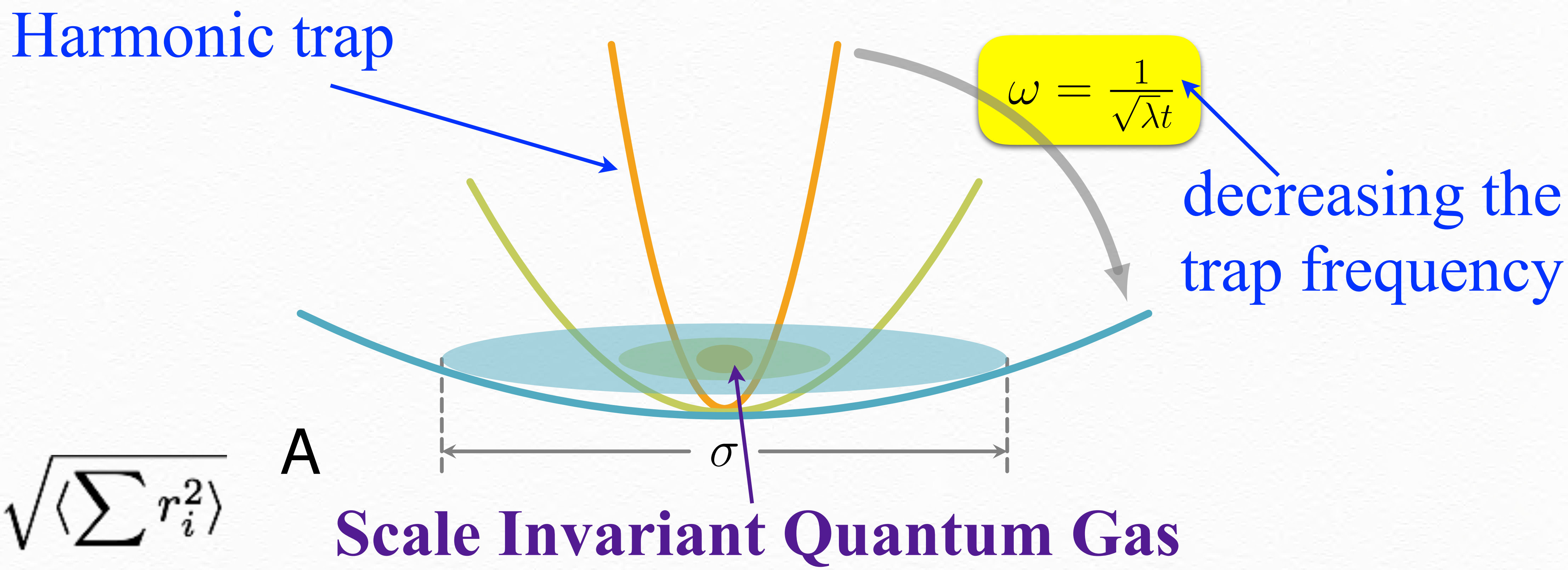


$$R = \sqrt{\langle \sum r_i^2 \rangle}$$

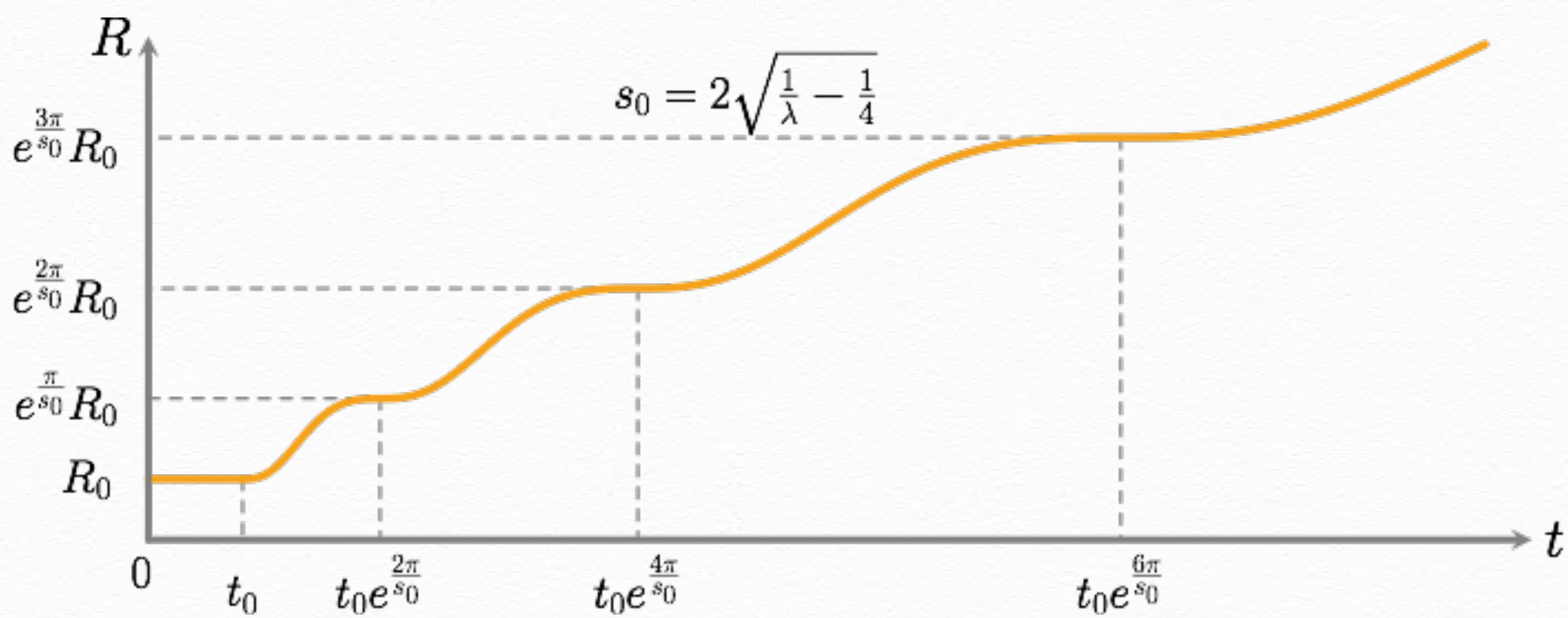
A



Expanding Harmonic Trap



$$R = \sqrt{\langle \sum r_i^2 \rangle}$$



Scale Invariance

$$i\hbar \frac{\partial}{\partial t}\Psi = -\sum_i \frac{\hbar^2}{2m}\nabla_i^2 \Psi$$

Scale Transformation

$$\boxed{\begin{aligned} \mathbf{r}_i &\longrightarrow \Lambda \mathbf{r}_i \\ t &\longrightarrow \Lambda^2 t \end{aligned}}$$

$$\frac{1}{\Lambda^2} \qquad\qquad \frac{1}{\Lambda^2}$$

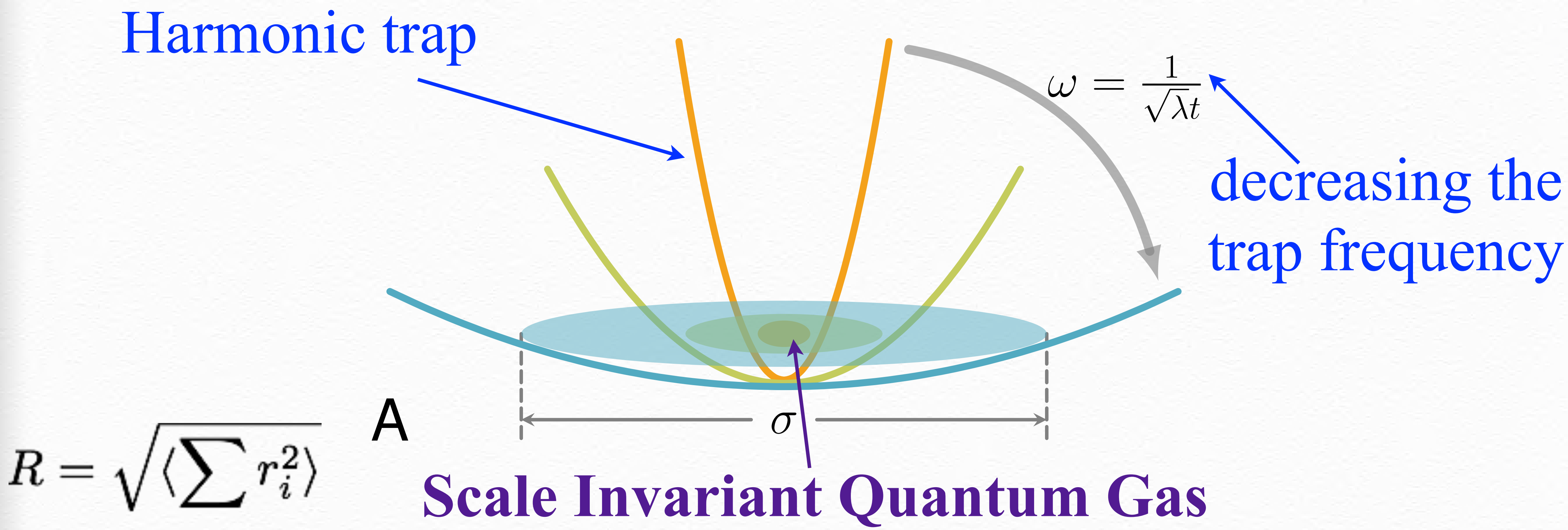
No other energy scale except for the kinetic energy

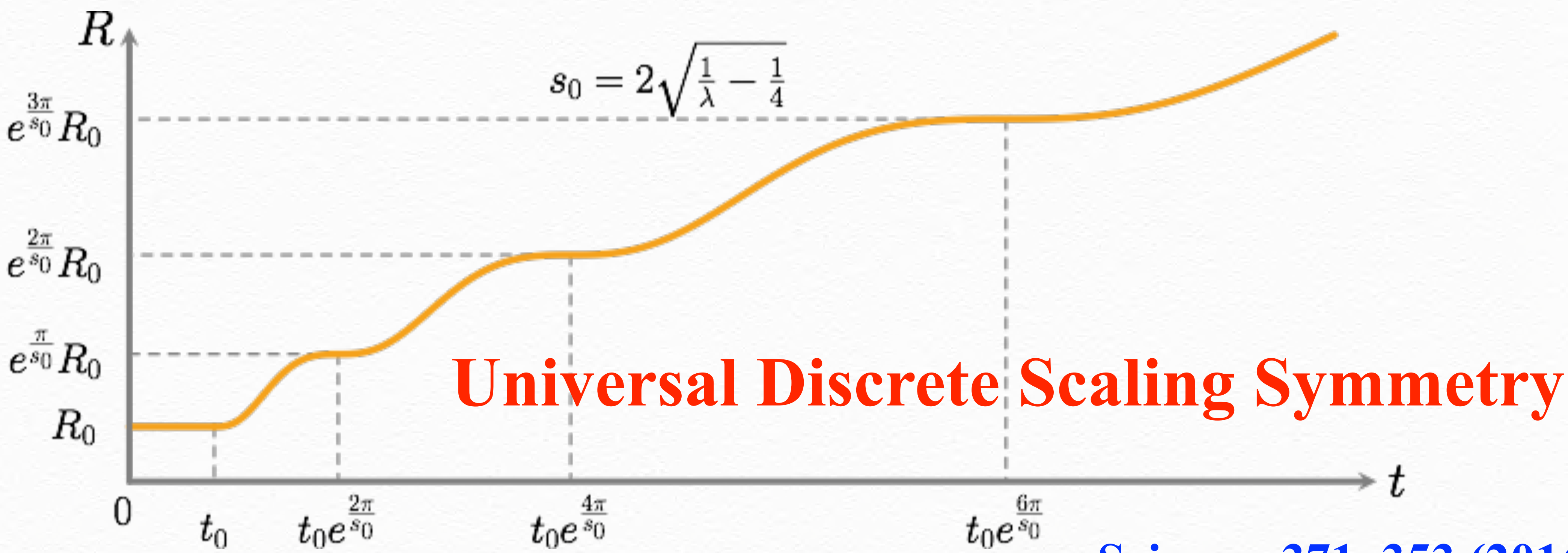
Zoo of Scale Invariant Quantum Gases

Non-interacting bosons/ fermions at any dimension	No other length scale except for density
Unitary Fermi gas at three dimension	Density and a_s $a_s = \infty$
Tonks gas of bosons/ fermions at one dimension	Density and g_{1D} $g_{1D} = \infty$

Universal behavior: $\boxed{\langle V \rangle = \alpha \langle T \rangle}$

Universal Discrete Scaling Symmetry



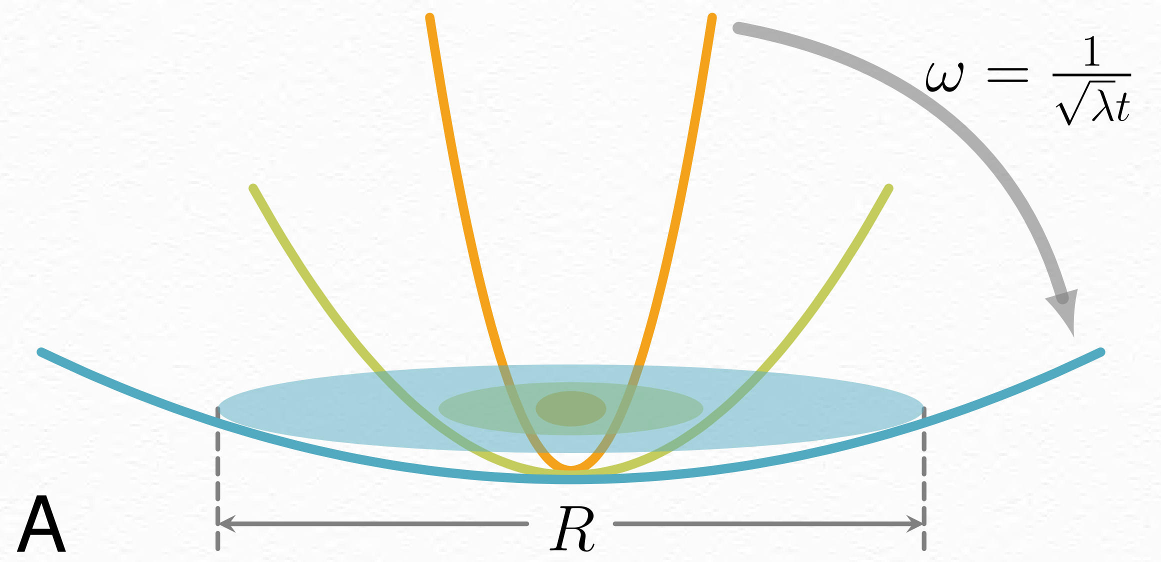


Science, 371, 353 (2016)

Universal Phenomena

- Universal
  - Independent of Temperature
  - Independent of State of Matter
  - Independent Dimension

Scaling Symmetry in a Harmonic Trap



$$i\hbar\frac{\partial}{\partial t}\Psi = \left[H + \sum_i \frac{1}{2} m\omega^2 r_i^2\right]\Psi$$

Scale Transformation

$$\mathbf{r}_i \longrightarrow \Lambda \mathbf{r}_i$$
$$t \longrightarrow \Lambda^2 t$$

$$\frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2}$$

This scaling symmetry exists only if $\boxed{\omega = \frac{1}{\sqrt{\lambda} t}}$

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

$$\frac{1}{2}\sum_i (\mathbf{r}_i \cdot \mathbf{p}_i + \mathbf{p}_i \cdot \mathbf{r}_i)$$

Generator of spatial scaling transformation

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H]\rangle = 2i\langle \hat{D}\rangle$$

$$i\frac{d}{dt}\langle \hat{D}\rangle = \langle [\hat{D}, H]\rangle = 2i\Big(\langle H\rangle - \omega^2 R^2\Big)$$

$$\frac{d}{dt}\langle H\rangle = \langle \frac{\partial}{\partial t} H\rangle = \omega\dot{\omega}R^2$$

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

$$i\frac{d}{dt}\langle \hat{D} \rangle = \langle [\hat{D}, H] \rangle = 2i\Big(\langle H \rangle - \omega^2 R^2 \Big)$$

$$\frac{d}{dt}\langle H \rangle = \langle \frac{\partial}{\partial t} H \rangle = \omega\dot{\omega} R^2$$

$$\boxed{\frac{d^3}{dt^3}R^2 + 4\omega^2 \frac{d}{dt}R^2 + 4\omega\dot{\omega}R^2 = 0}$$

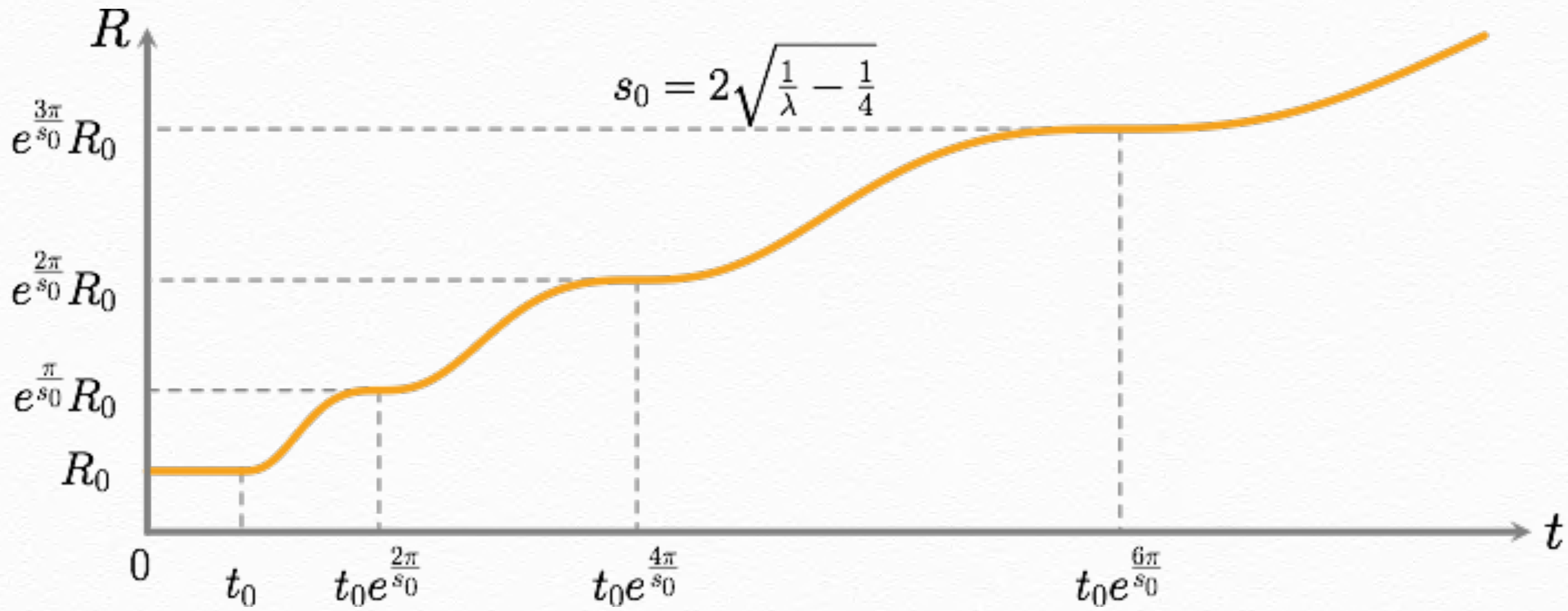
$$\omega \sim \frac{1}{t} \qquad \frac{1}{t^3} \qquad \frac{1}{t^3} \qquad \frac{1}{t^3}$$

Scaling Symmetry in Time: $t \rightarrow \lambda t$

Expansion Dynamics

Boundary Condition Breaks the Scaling Symmetry to a Discrete One:

$$\frac{\langle \hat{R}^2\rangle(t)}{R_0^2} = \frac{t}{t_0}\frac{1}{\sin^2\varphi}\left[1-\cos\varphi\cdot\cos\left(s_0\ln\frac{t}{t_0}+\varphi\right)\right]$$

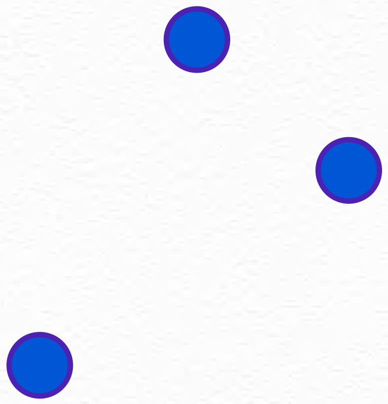


Why plateaus ?

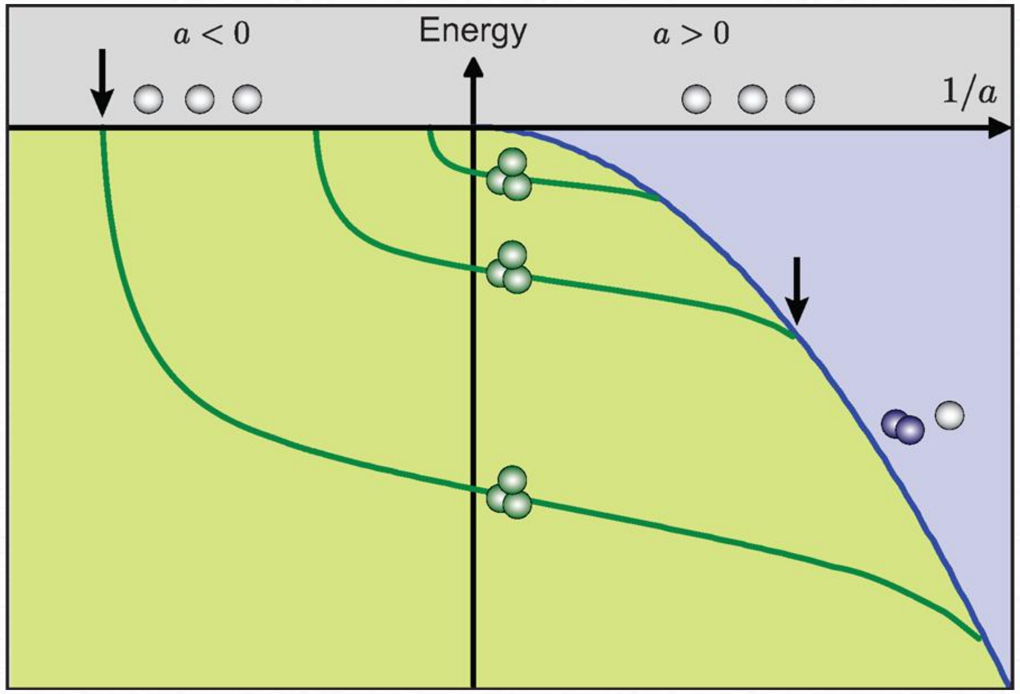
$$\frac{d^n}{dt^n}\langle \hat{R}^2\rangle|_{t=t_0} = 0$$

The Efimov Effect

Problem: Three bosons interacting through a short-range interaction



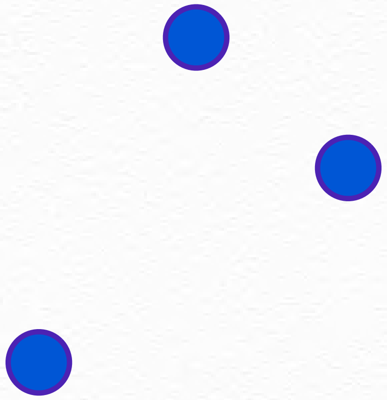
1970



Universal Discrete Scaling Symmetry

The Efimov Effect

Problem: Three bosons interacting through a short-range interaction



1970

$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2} - \frac{s_0^2 + 1/4}{m\rho^2}\right]\psi = E\psi$$

$$\boxed{\psi = \sqrt{\rho}\cos[s_0 \log(\rho/\rho_0)]}$$

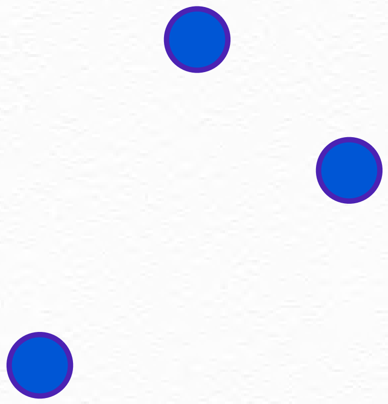
$$\rho \longrightarrow e^{2\pi/s_0}\rho$$

$$E_{\mathrm{T}}^{(n+1)}/E_{\mathrm{T}}^{(n)} \simeq e^{-2\pi/s_0}$$

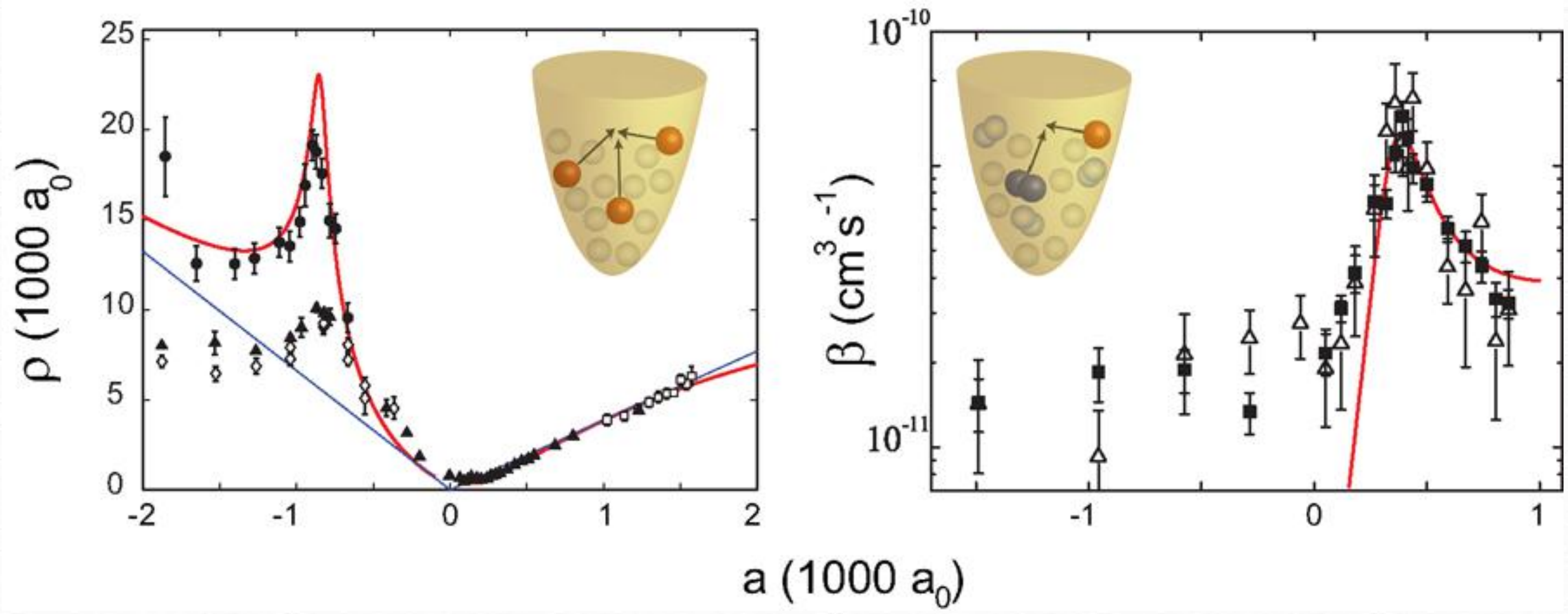
Discrete Scaling Symmetry

The Efimov Effect

Problem: Three bosons interacting through a short-range interaction



1970



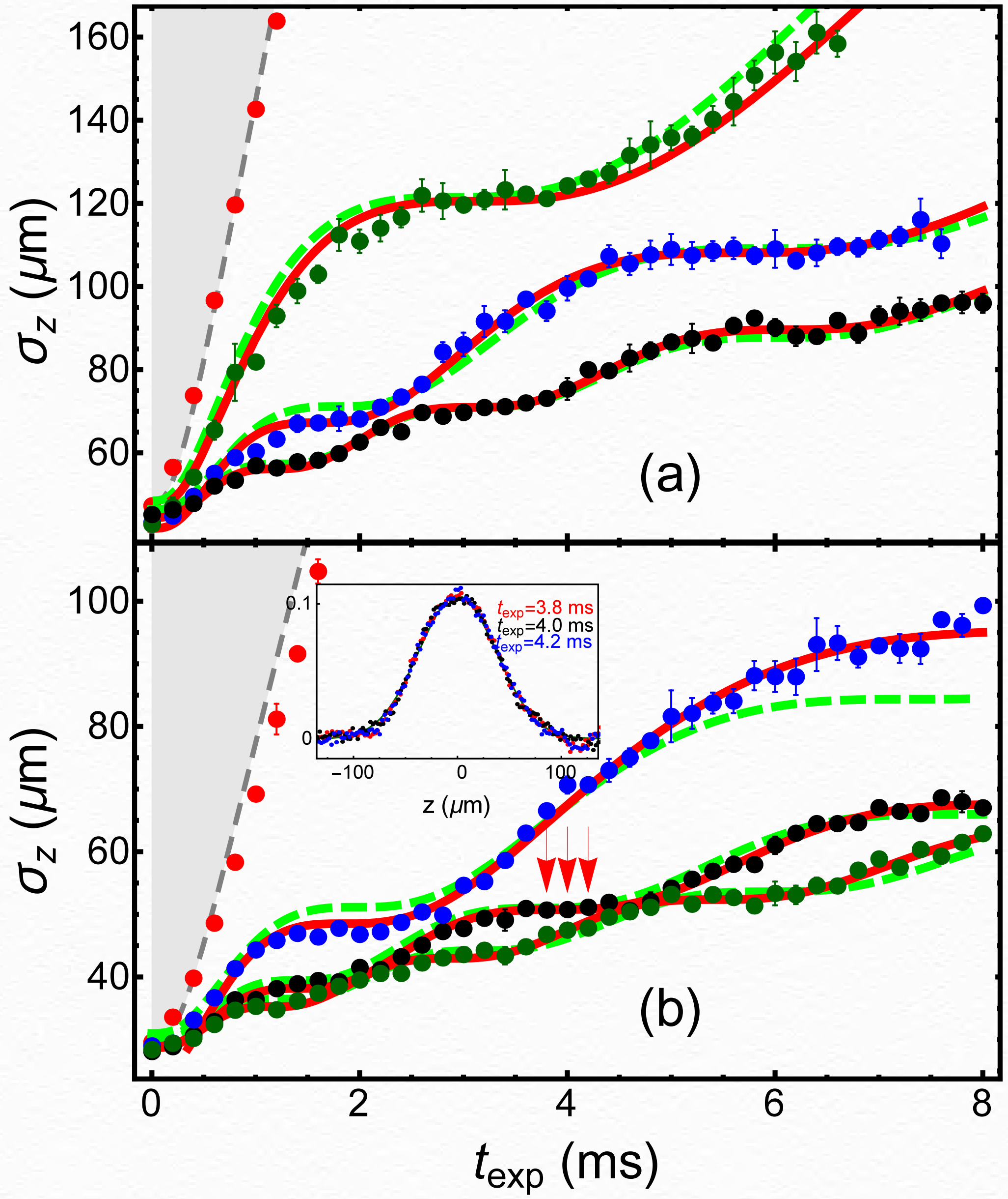
Innsbruck 2005, and many later

Connection to the Efimov Effect

The Efimov Effect	The "Efimovian" Expansion
$-\frac{\hbar^2 d^2}{2md^2\rho}\psi - \frac{\lambda}{\rho^2}\psi = E\psi$	$\frac{d^3}{dt^3}\langle\hat{R}^2\rangle + \frac{4}{\lambda t^2}\frac{d}{dt}\langle\hat{R}^2\rangle - \frac{4}{\lambda t^3}\langle\hat{R}^2\rangle = 0.$
Spatial continuous scaling symmetry	Temporal continuous scaling symmetry
Short-range boundary condition	Initial time
$\psi = \sqrt{\rho}\cos[s_0\log(\rho/\rho_0)]$	$\frac{\langle\hat{R}^2\rangle(t)}{R_0^2} = \frac{t}{t_0}\frac{1}{\sin^2\varphi}\left[1 - \cos\varphi\cdot\cos\left(s_0\ln\frac{t}{t_0} + \varphi\right)\right]$
Spatial discrete scaling symmetry $\rho \longrightarrow e^{2\pi/s_0}\rho$	Temporal discrete scaling symmetry $t \longrightarrow e^{2\pi/s_0}t$

Experimental Observation

by Haibin Wu in East China Normal University



Non-interacting

Unitary Fermions

$$\frac{\langle \hat{R}^2\rangle(t)}{R_0^2} = \frac{t}{t_0}\frac{1}{\sin^2\varphi}\left[1-\cos\varphi\cdot\cos\left(s_0\ln\frac{t}{t_0}+\varphi\right)\right]$$

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Experimental Observation

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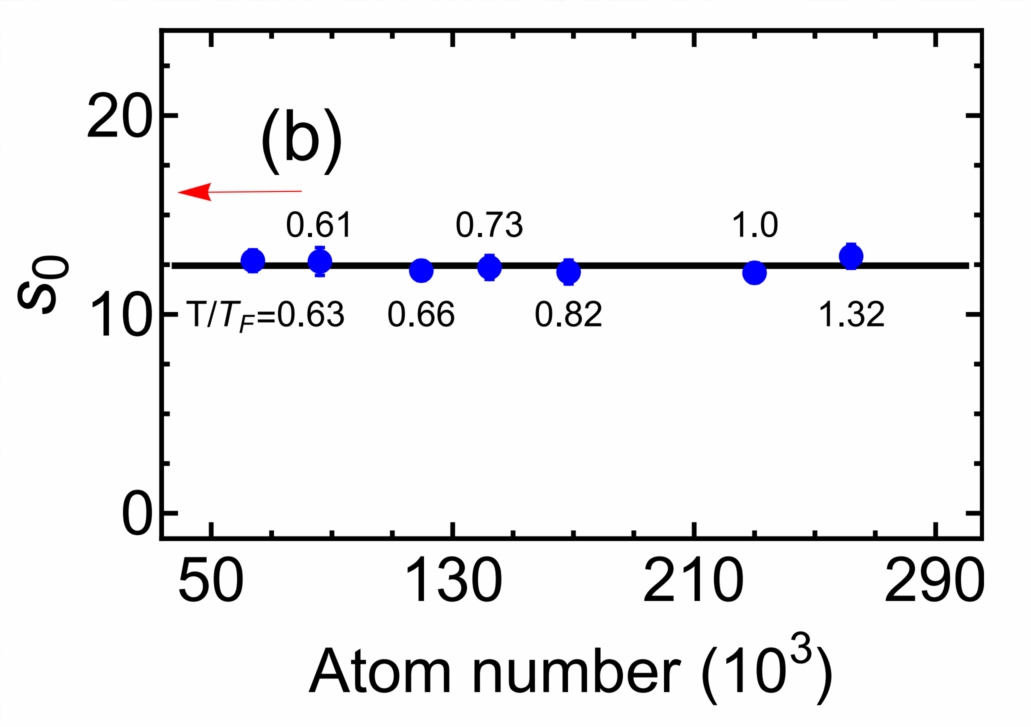

Independent of Temperature

Independent of State of Matter

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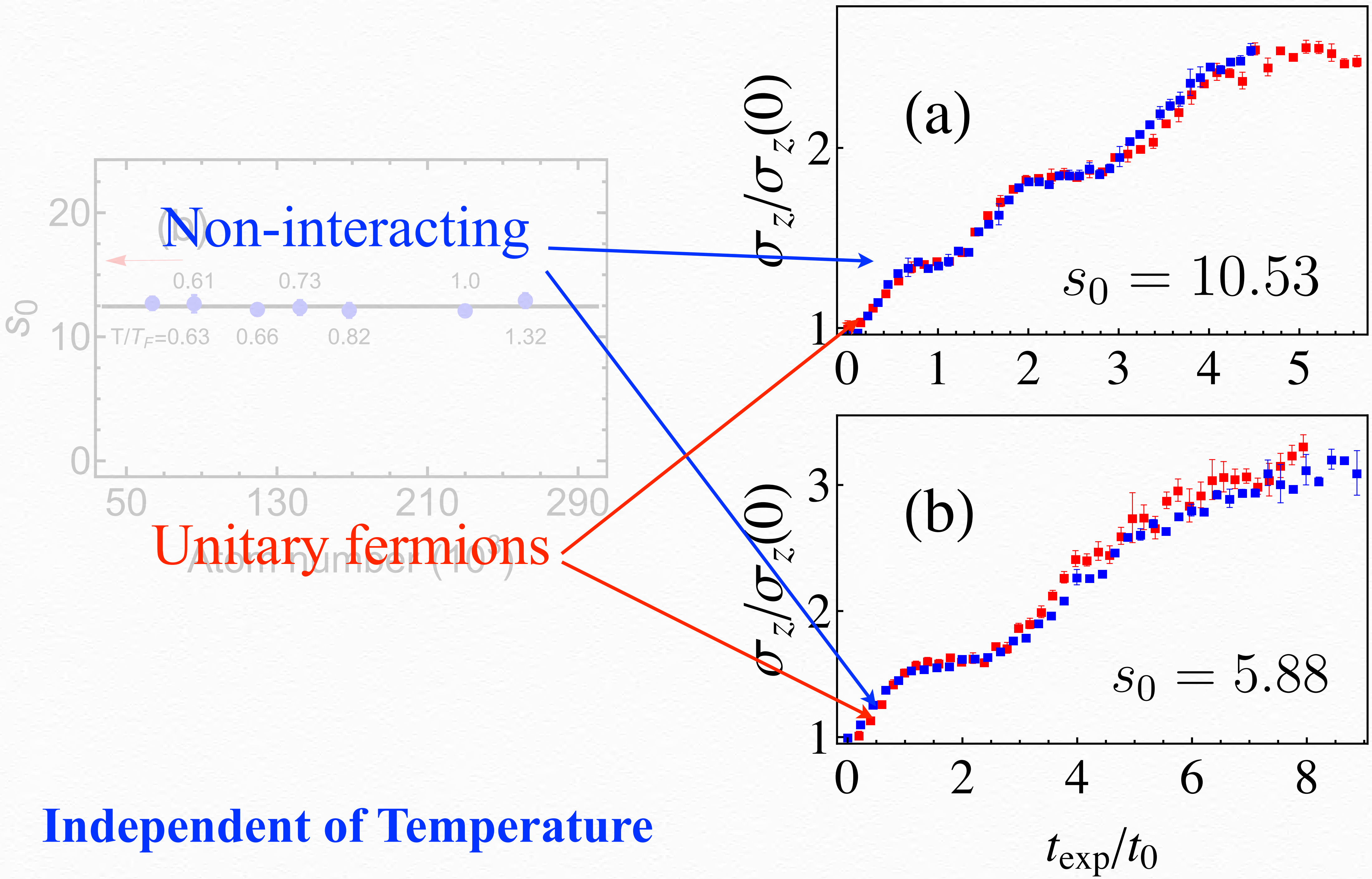


Independent of Temperature

Independent of State of Matter

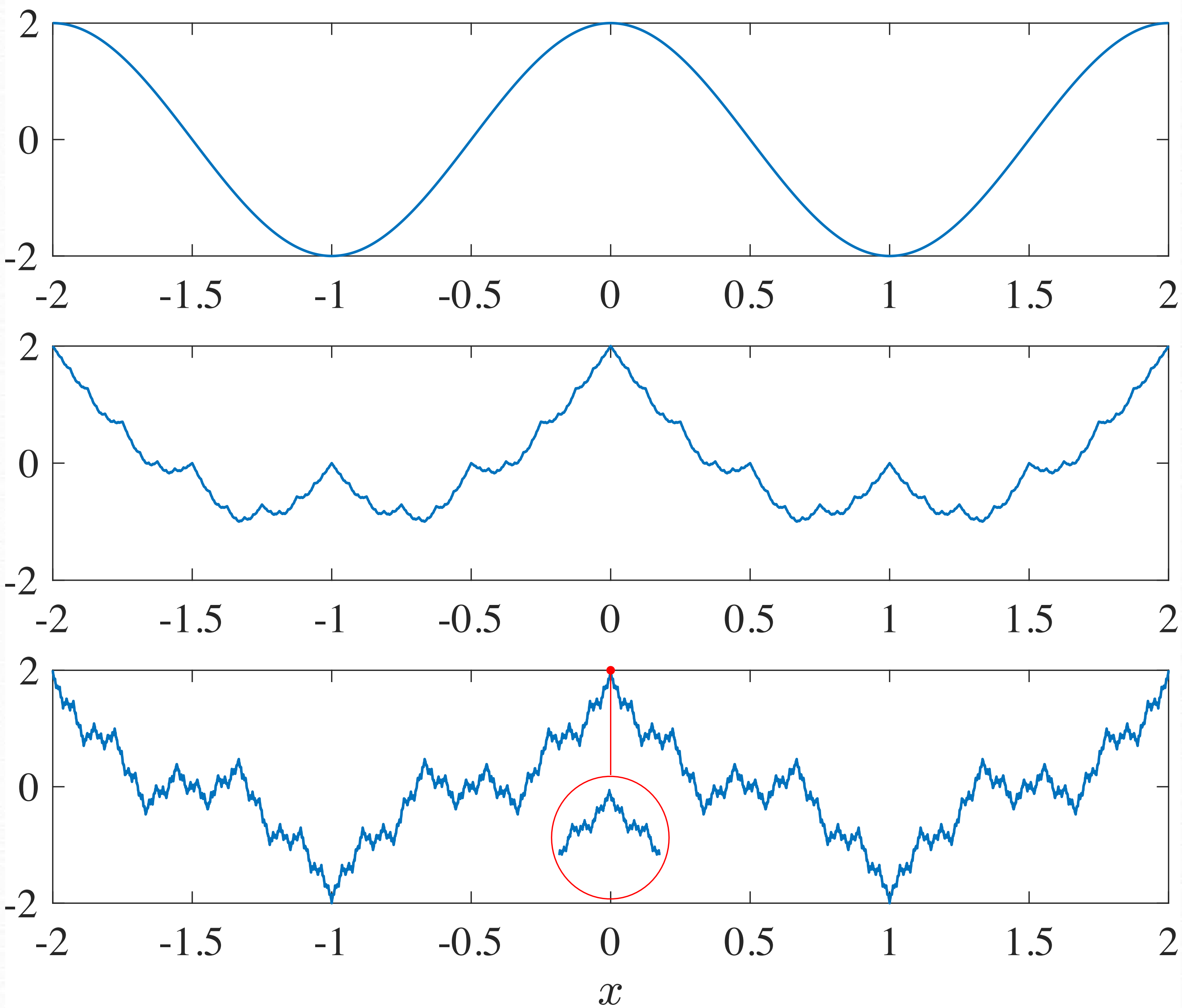
Science, 371, 353 (2016)

Fractal: Weierstrass Functions

$ab < 1$

$ab = 1$

$ab > 1$



$$W(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x).$$

Eigen-Energy with Scaling Symmetry

$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2} - \frac{s_0^2 + 1/4}{m\rho^2}\right]\psi = E\psi$$

$\rho \to \lambda\rho$

$E \to \frac{E}{\lambda^2}$

The Equation is Invariant

Eigen-Energy with Scaling Symmetry

$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2} - \frac{s_0^2 + 1/4}{m\rho^2}\right]\psi = E\psi$$

$\rho \to \lambda\rho$

$E \to \frac{E}{\lambda^2}$

The Equation is Invariant

$$\boxed{\psi_{n+1}(\mathbf{r}) \propto \psi_n(\lambda\mathbf{r})}$$

$$\boxed{E_{n+1} \simeq \lambda^2 E_n}$$

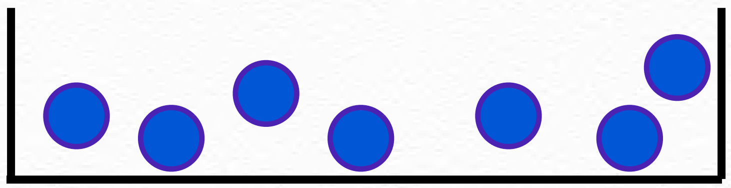
$$W(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x).$$

Chao Gao, Hui Zhai, Zheyu Shi, PRL, 2019

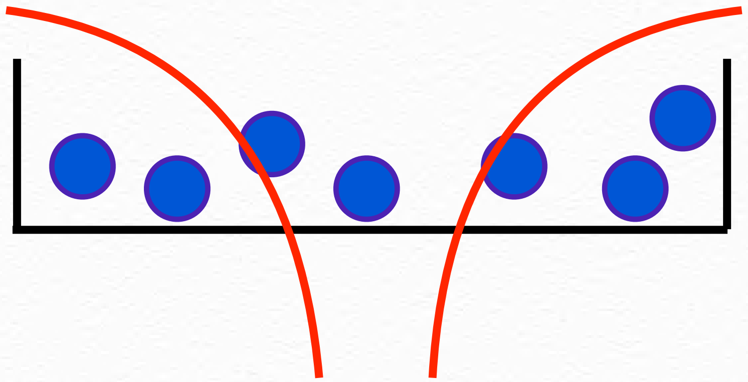
Dynamical Fractal from Quench Dynamics

Potential Quench:

$$V(x) = -\frac{\hbar^2}{2m}\frac{s_0^2 + 1/4}{x^2 + r_0^2}$$



$t = 0$



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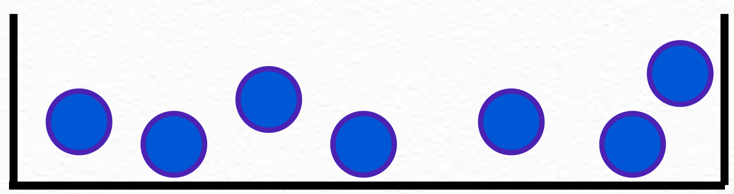

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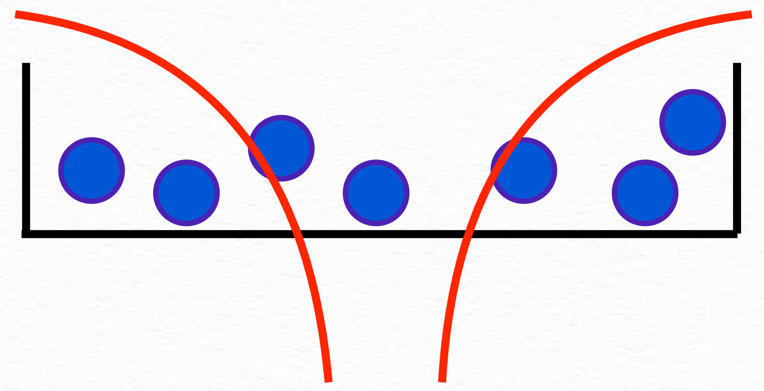
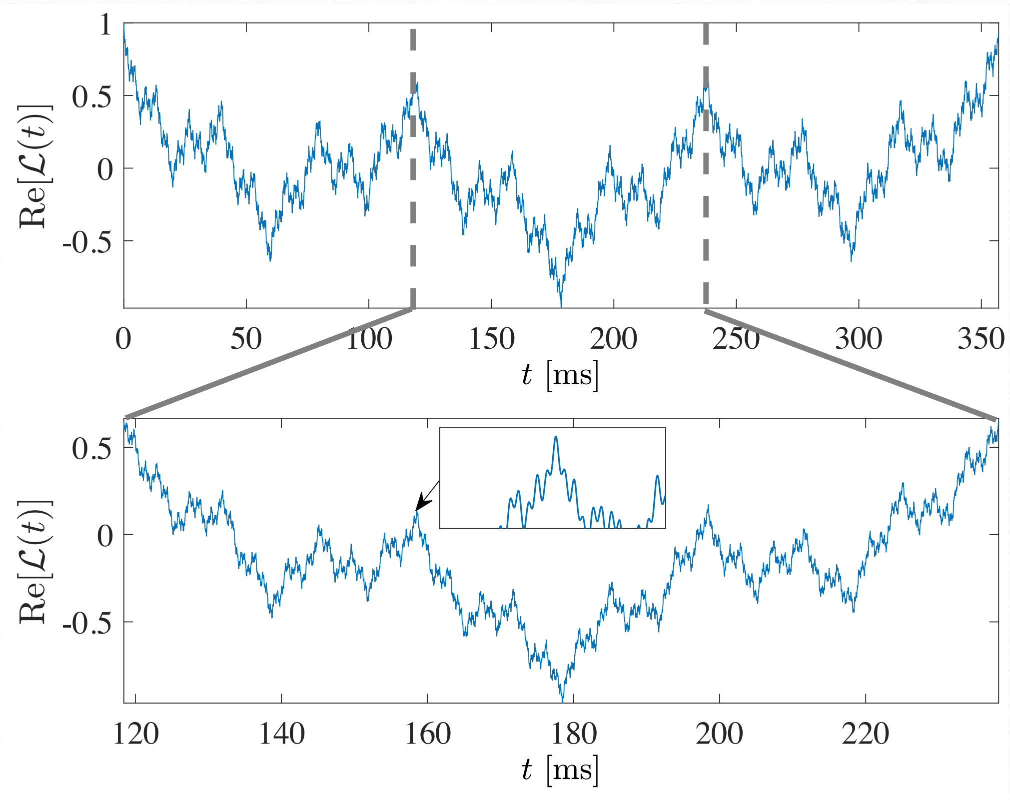
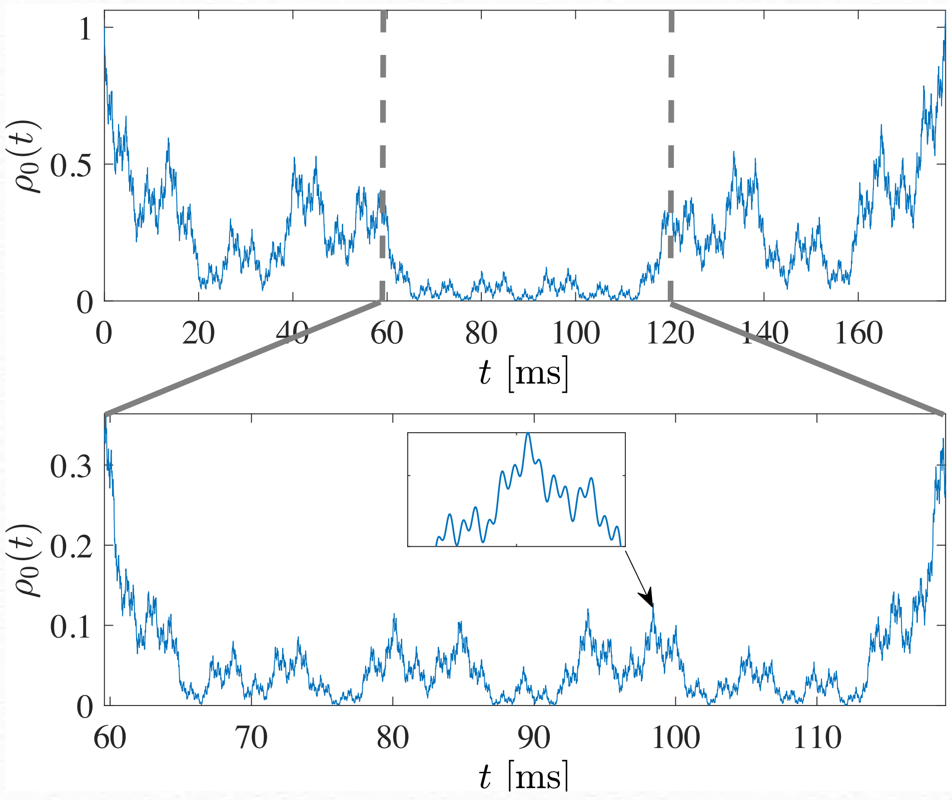






Loschmidt Echo

Zero-momentum Distribution

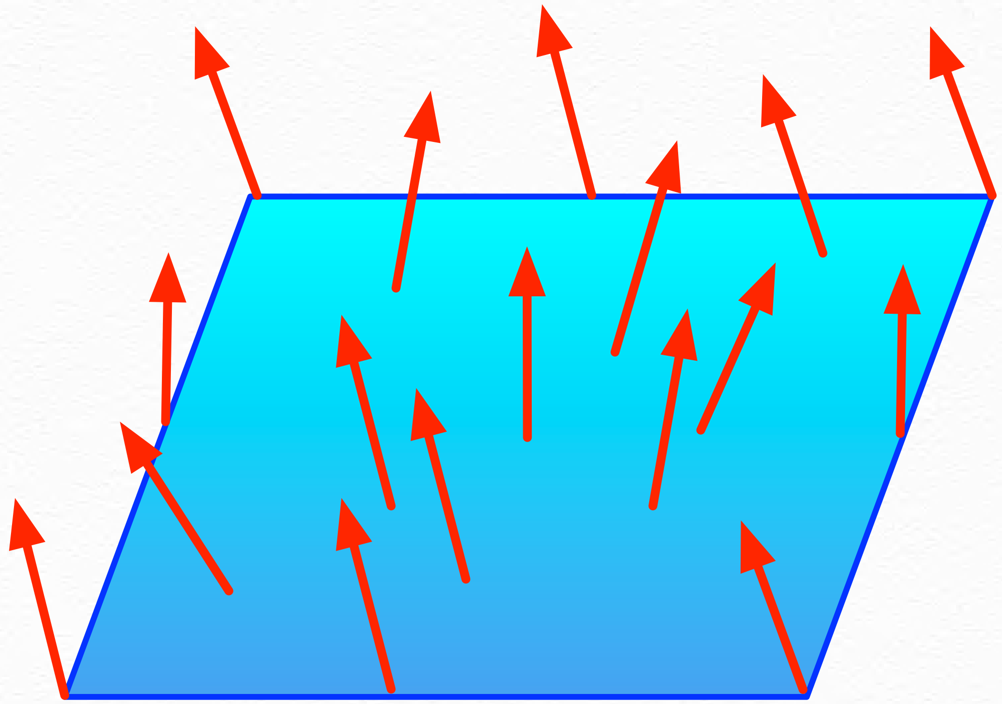
Chao Gao, Hui Zhai, Zheyu Shi, PRL, 2019

Topology

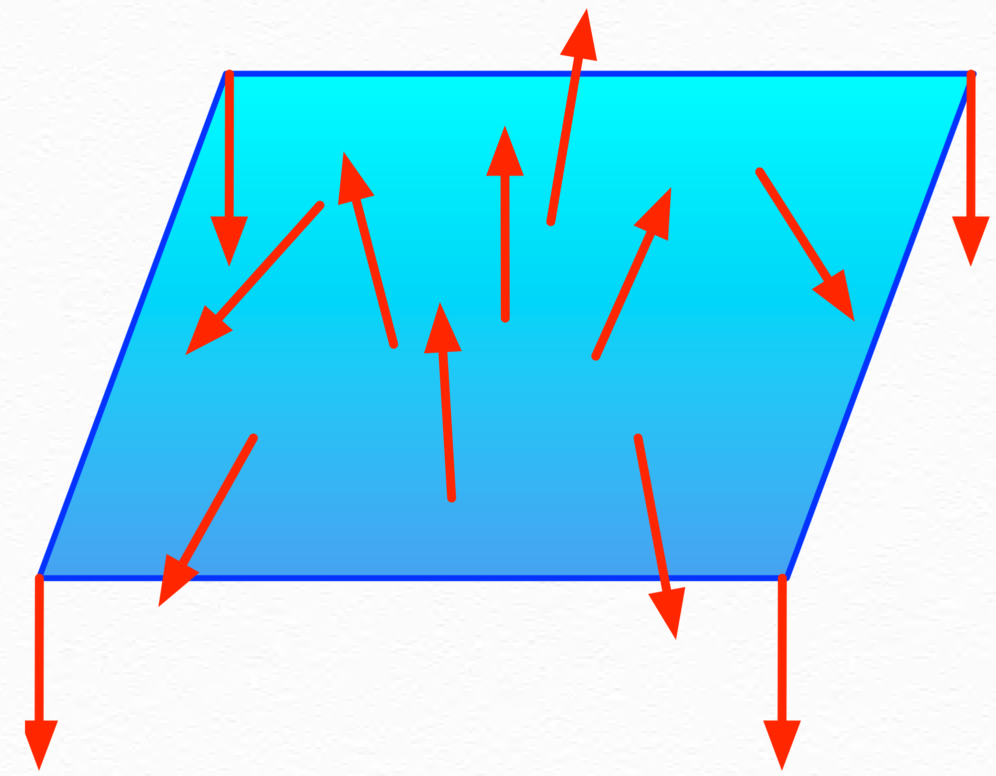
Topological Band Theory

$$\hat{\mathcal{H}} = \sum_{\mathbf{k}} (\hat{c}^\dagger_{\uparrow,\mathbf{k}}, \hat{c}^\dagger_{\downarrow,\mathbf{k}}) H_{\mathbf{k}} \begin{pmatrix} \hat{c}_{\uparrow,\mathbf{k}} \\ \hat{c}_{\downarrow,\mathbf{k}} \end{pmatrix}$$

$$\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$



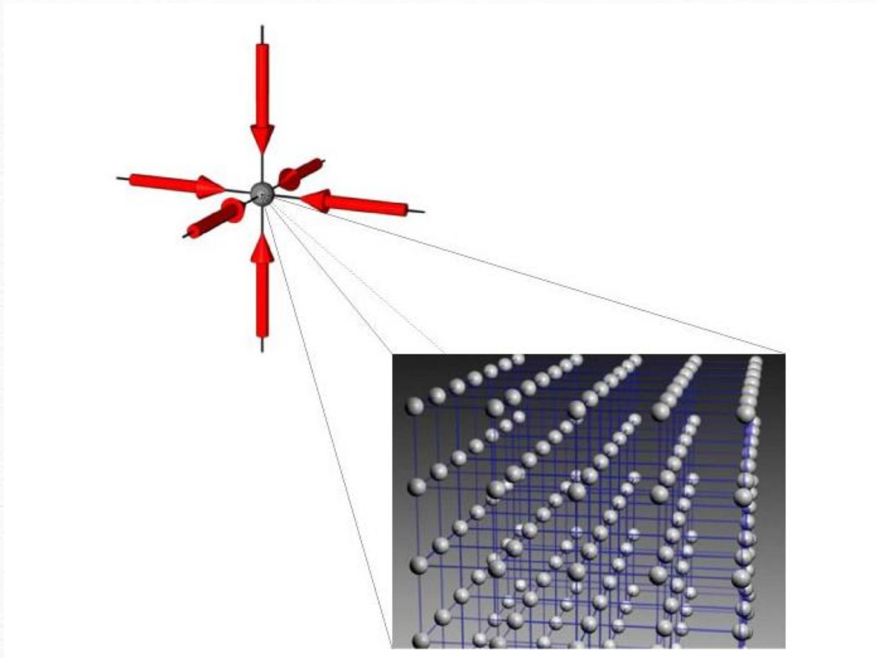
Topological Trivial



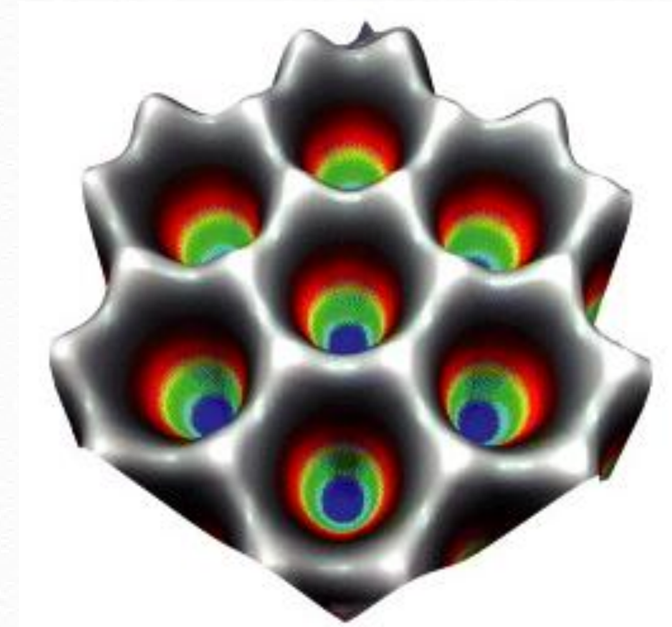
Topological Non-trivial

$$\Pi_2(S^2) = Z$$

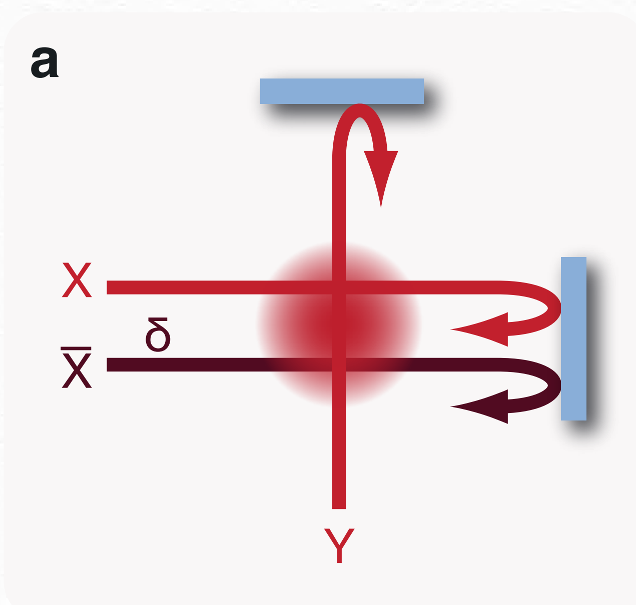
Optical Lattice

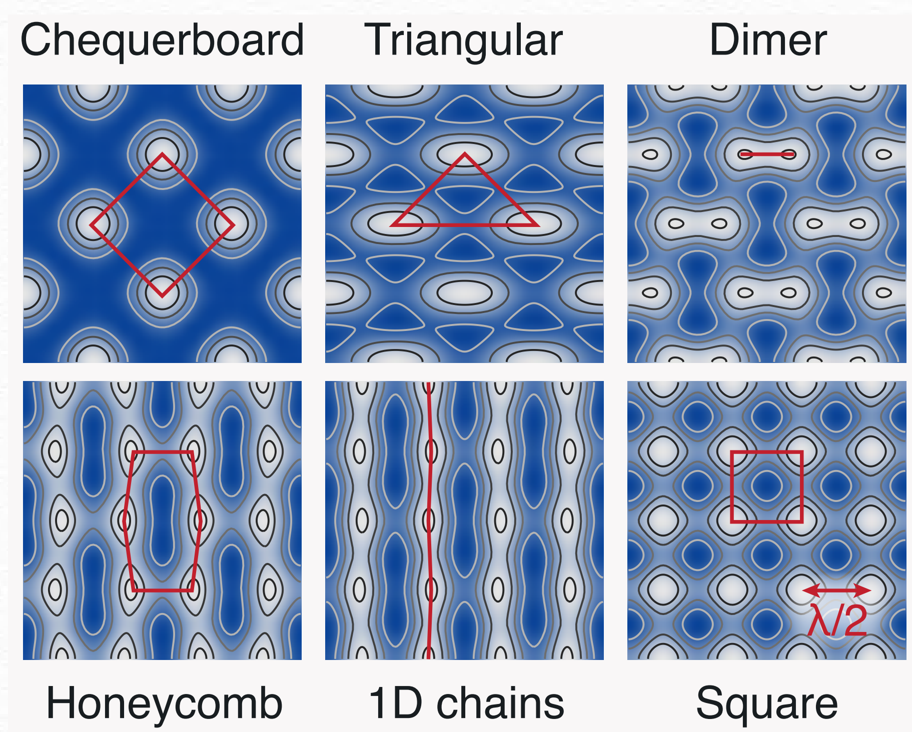


Cubic Lattice



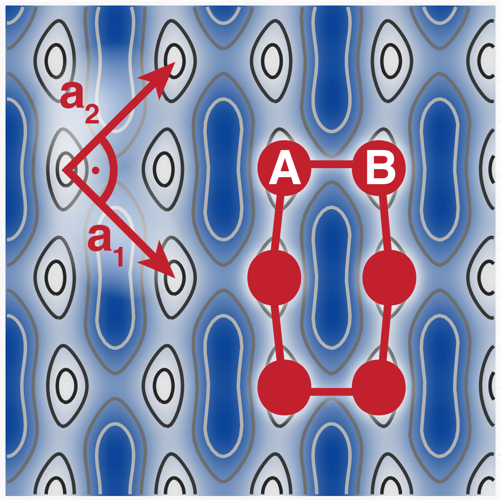
Triangular Lattice

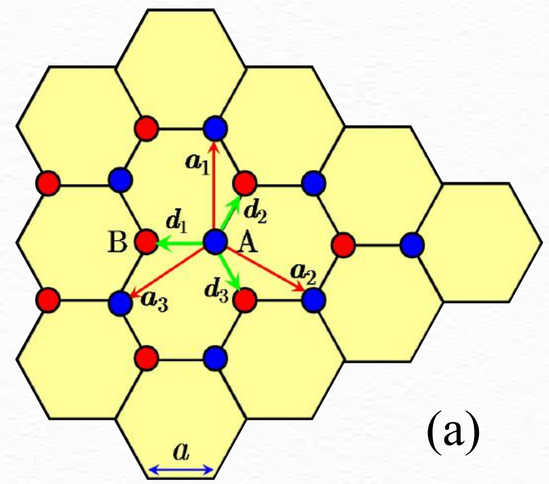




Tunable Geometry (ETH, 2012)

Dirac Point: Gapless

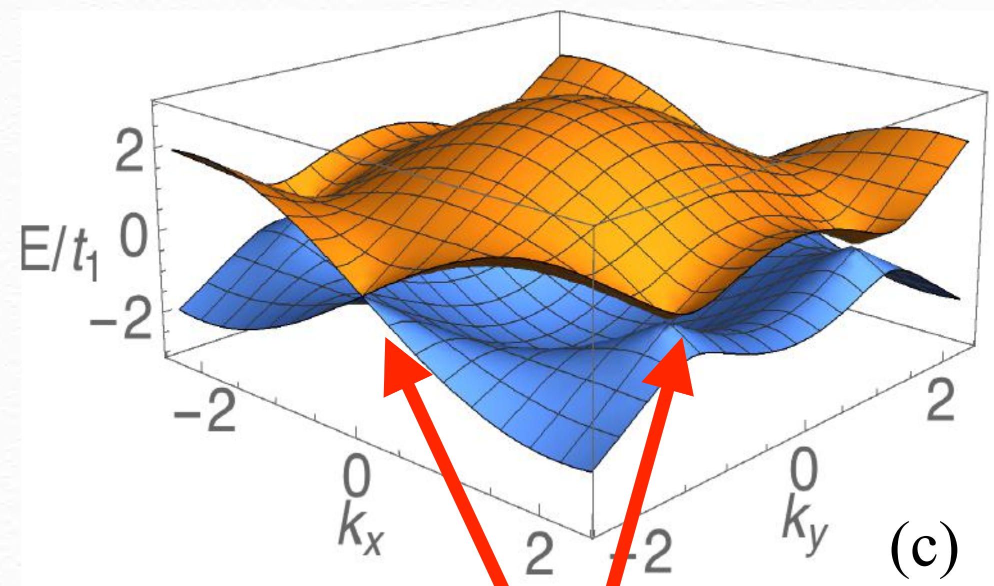




Honeycomb lattice

$$\hat{H} = -t_1 \sum_{\langle ij \rangle} \left(\hat{c}^\dagger_{\mathrm{B},j} \hat{c}_{\mathrm{A},i} + \mathrm{h.c.} \right)$$

Dirac Point



$$B_x = 0$$
$$B_y = 0$$

$$\hat{H} = \sum_{\mathbf{k}} \left(\hat{c}^\dagger_{\mathrm{A}}(\mathbf{k}), \hat{c}^\dagger_{\mathrm{B}}(\mathbf{k}) \right) H(\mathbf{k}) \begin{pmatrix} \hat{c}_{\mathrm{A}}(\mathbf{k}) \\ \hat{c}_{\mathrm{B}}(\mathbf{k}) \end{pmatrix}$$

$$H(\mathbf{k}) = \begin{pmatrix} 0 & -t_1 \sum_\alpha e^{-i\mathbf{k}\cdot\mathbf{d}_\alpha} \\ -t_1 \sum_\alpha e^{i\mathbf{k}\cdot\mathbf{d}_\alpha} & 0 \end{pmatrix}$$

$$H(\mathbf{k}) = \mathbf{B}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

$$B_x(\mathbf{k}) = -t_1 \sum_\alpha \cos(\mathbf{k}\cdot\mathbf{d}_\alpha)\,; B_y(\mathbf{k}) = -t_1 \sum_\alpha \sin(\mathbf{k}\cdot\mathbf{d}_\alpha)$$

From Dirac Point to Haldane Model

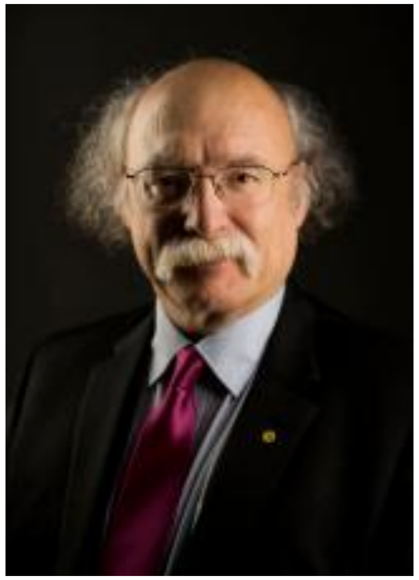
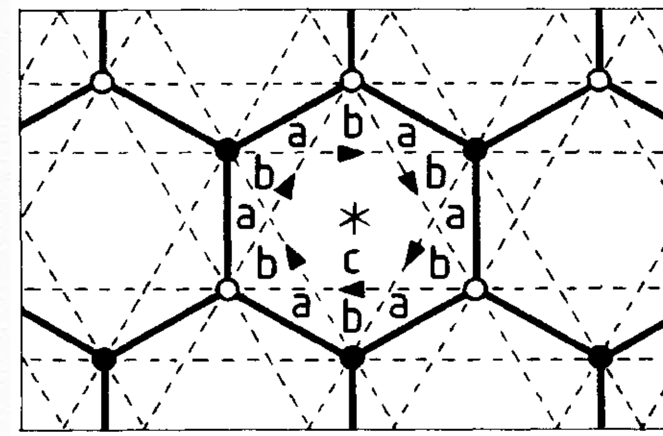


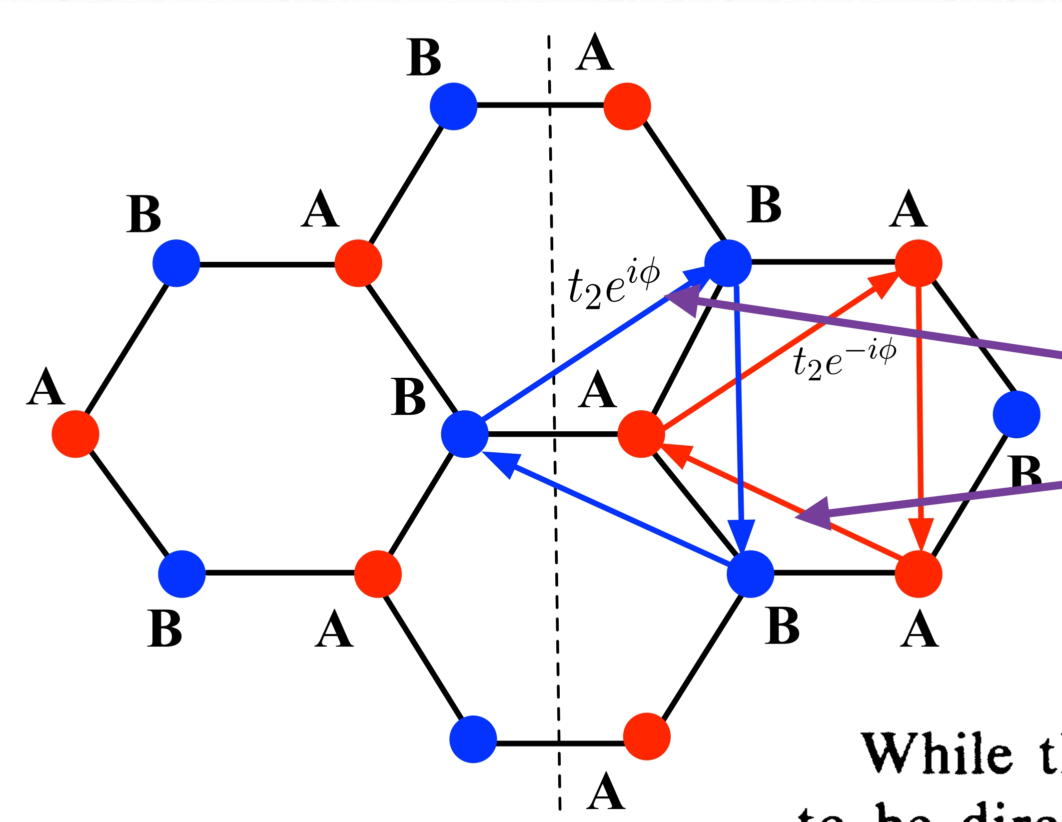
Photo: A. Mahmoud
F. Duncan M. Haldane
Prize share: 1/4

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane
Department of Physics, University of California, San Diego, La Jolla, California 92093
(Received 16 September 1987)





How to realize this nontrivial next-nearest hopping ??

While the particular model presented here is unlikely to be directly physically realizable, it indicates that, at

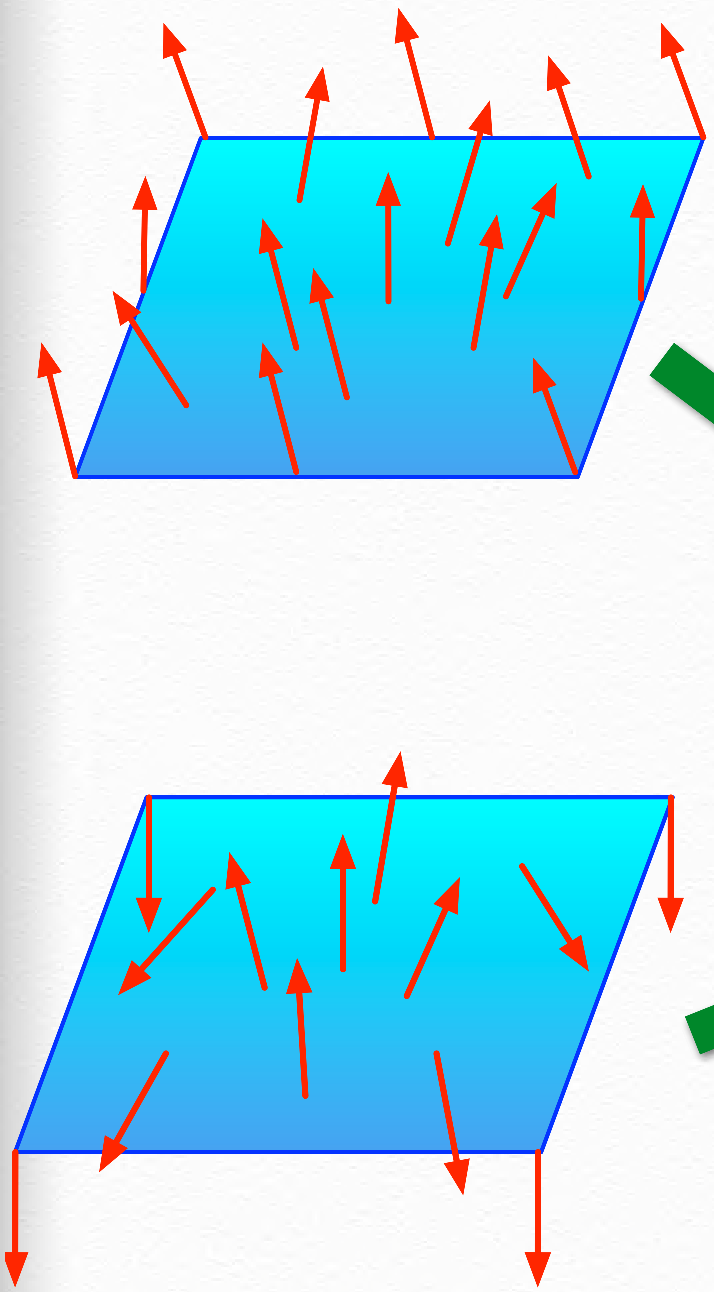
Haldane Model

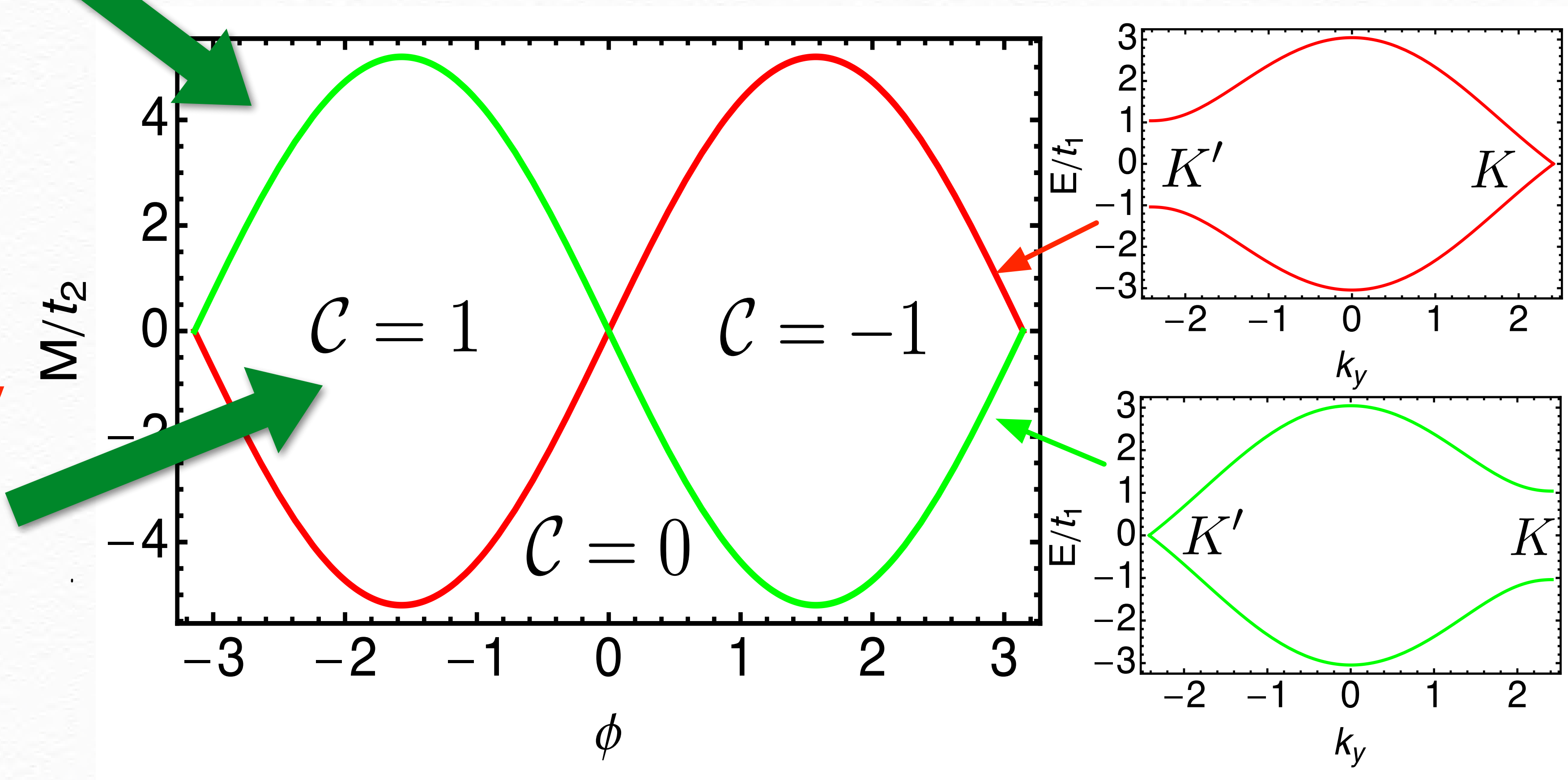
$$\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

$$h_x = -J_1\left[\cos k_x + \cos\left(\frac{k_x}{2} + \frac{\sqrt{3}k_y}{2}\right) + \cos\left(\frac{k_x}{2} - \frac{\sqrt{3}k_y}{2}\right)\right]$$

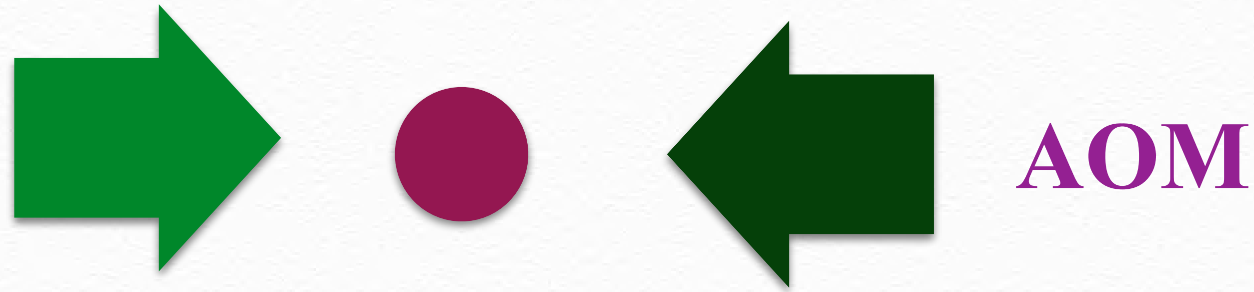
$$h_y = -J_2\left[\sin\left(\frac{k_x}{2} + \frac{\sqrt{3}k_y}{2}\right) + \sin\left(\frac{k_x}{2} - \frac{\sqrt{3}k_y}{2}\right)\right]$$

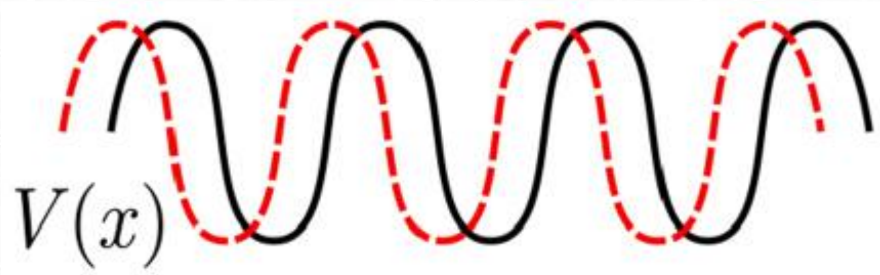
$$h_z = M + 2J_2\sin\phi\left[\sin\left(\sqrt{3}k_y\right) + \sin\left(\frac{3k_x}{2} - \frac{\sqrt{3}k_y}{2}\right) - \sin\left(\frac{3k_x}{2} + \frac{\sqrt{3}k_y}{2}\right)\right]$$





Shaking Optical Lattice





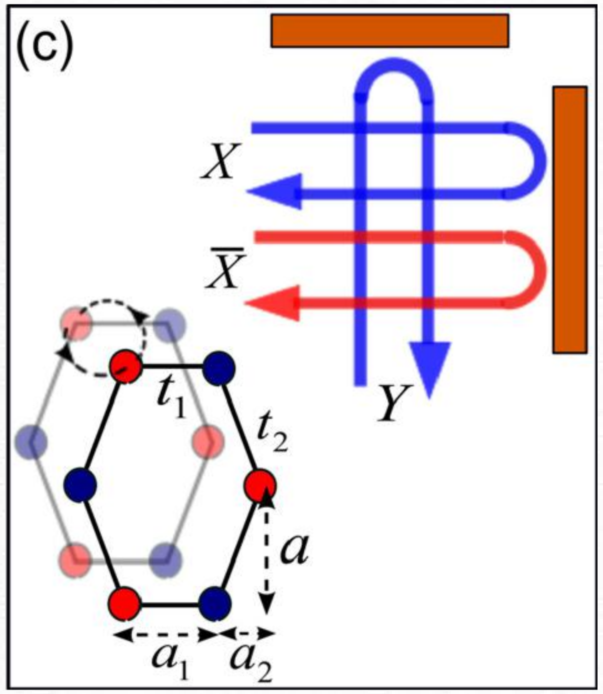
$$\hat{F} = \hat{U}\left(T_i + T, T_i\right) = \hat{\mathcal{T}}\,\exp\left\{-i\int_{T_i}^{T_i+T} dt\,\hat{H}\left(t\right)\right\}$$

For sufficiently fast modulation, if one only concerns the observation at integer period

$$\hat{F} = e^{-i\hat{H}_{\text{eff}}T}$$

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \sum_{n=1}^{\infty}\left\{\frac{[\hat{H}_n, \hat{H}_{-n}]}{n\omega} - \frac{[\hat{H}_n, \hat{H}_0]}{e^{-2\pi n i\alpha}n\omega} + \frac{[\hat{H}_{-n}, \hat{H}_0]}{e^{2\pi n i\alpha}n\omega}\right\}$$

Shaking Optical Lattice

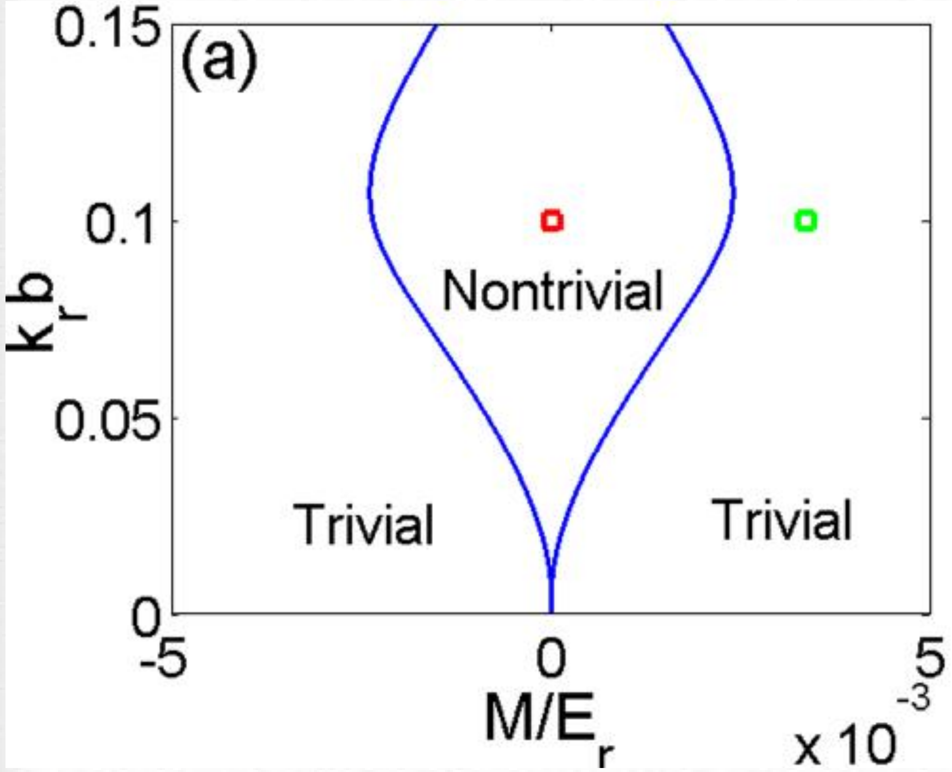


$$H = -\frac{\hbar^2\nabla^2}{2m} + V\left[x + b\sin\left(\omega t + \varphi\right), y + b\sin\left(\omega t\right)\right]$$

$$x' = x + b\cos\left(\omega t\right)$$
$$y' = y + b\sin\left(\omega t\right)$$

$$H\left(x,y,t\right) = \frac{\hbar^2}{2m}\left[-i\partial_x - A_x\left(t\right)\right]^2 + \frac{1}{2m}\left[-i\partial_y - A_y\left(t\right)\right]^2 + V\left(x,y\right)$$

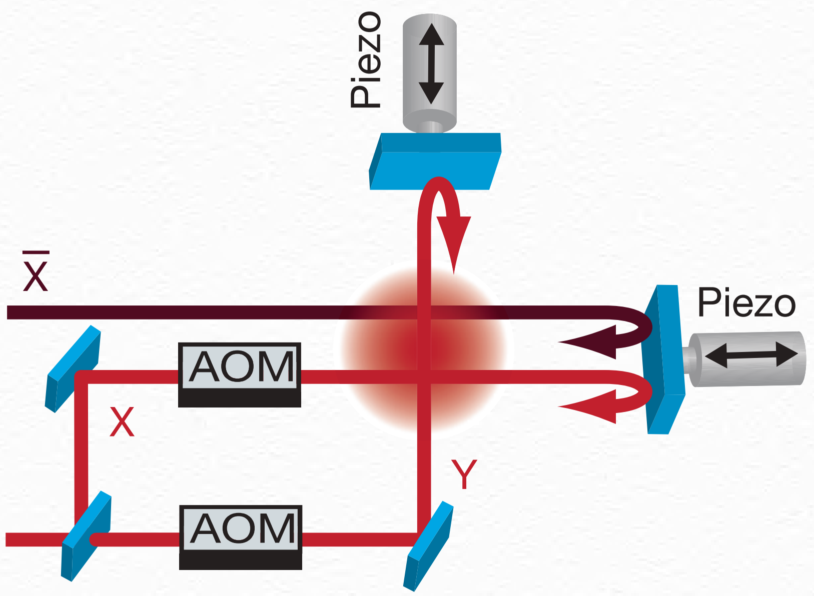
$$A_x\left(t\right) = m\omega b\sin\left(\omega t\right)/\hbar$$
$$A_y\left(t\right) = -m\omega b\cos\left(\omega t\right)/\hbar$$

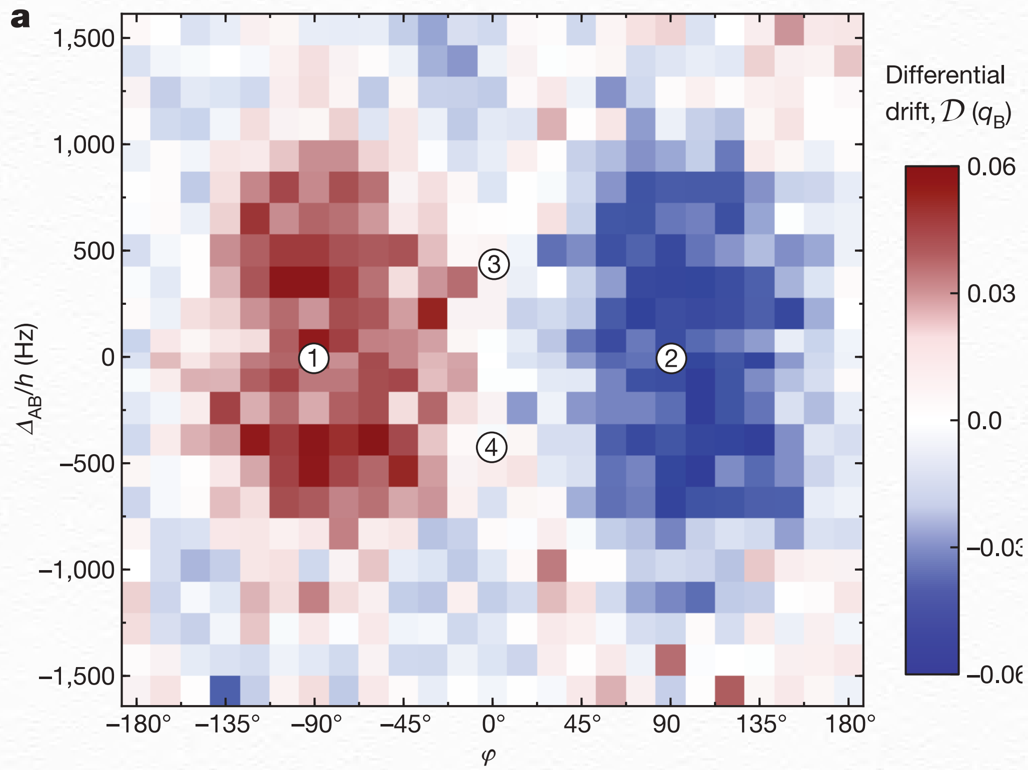


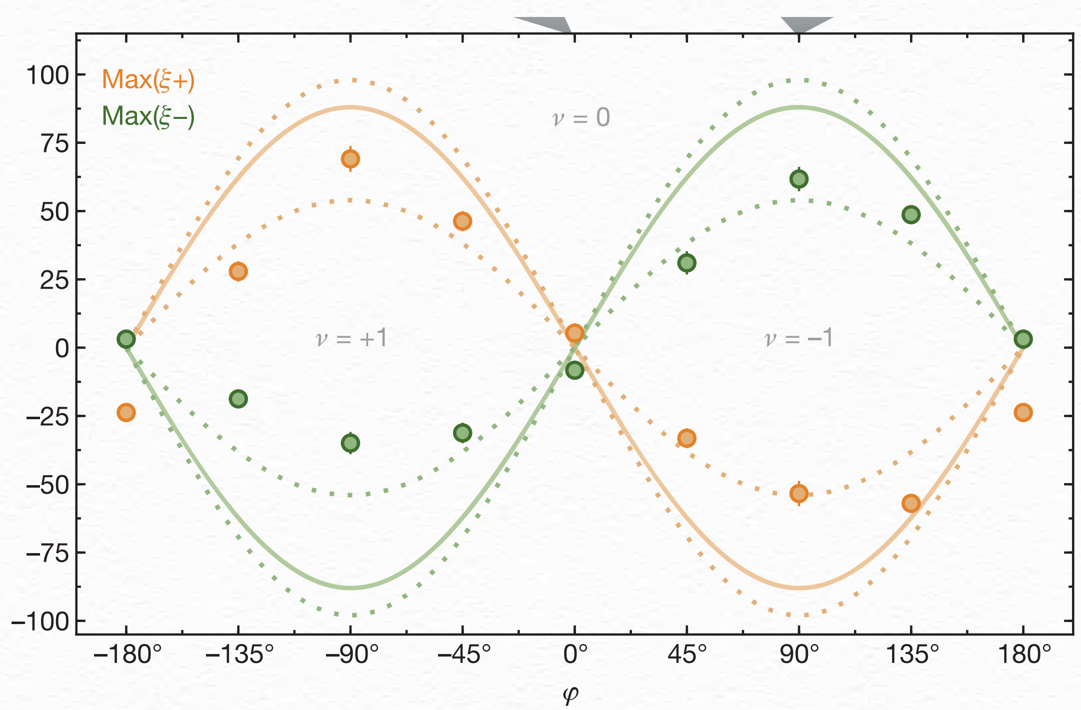
$$H_{\text{eff}}\left(\mathbf{k}\right) \approx H_0\left(\mathbf{k}\right) + \frac{\left[H_1\left(\mathbf{k}\right), H_{-1}\left(\mathbf{k}\right)\right]}{\omega}$$

Wei Zheng and Hui Zhai, PRA 2014

Experimental Realization







ETH, Nature (2014), See also Hamburg group, USTC group

Physical Consequence of 2D Chern Insulator

Physical Consequence of Topological Number

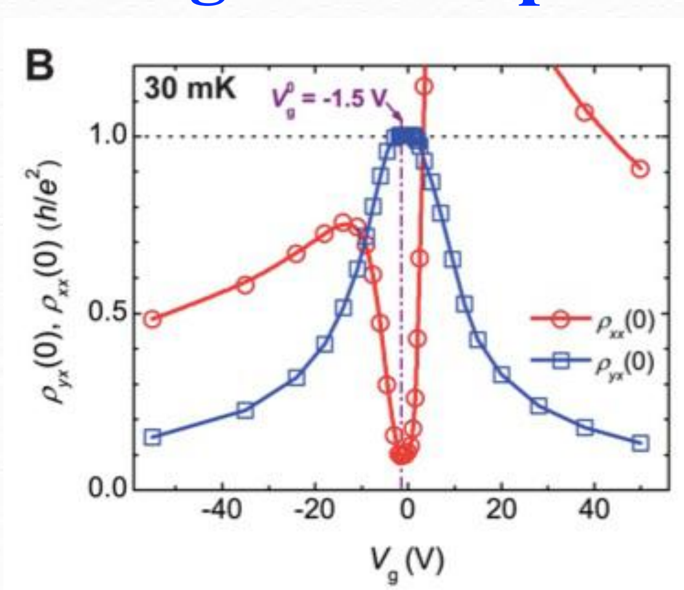
At or Near Equilibrium

From from Equilibrium

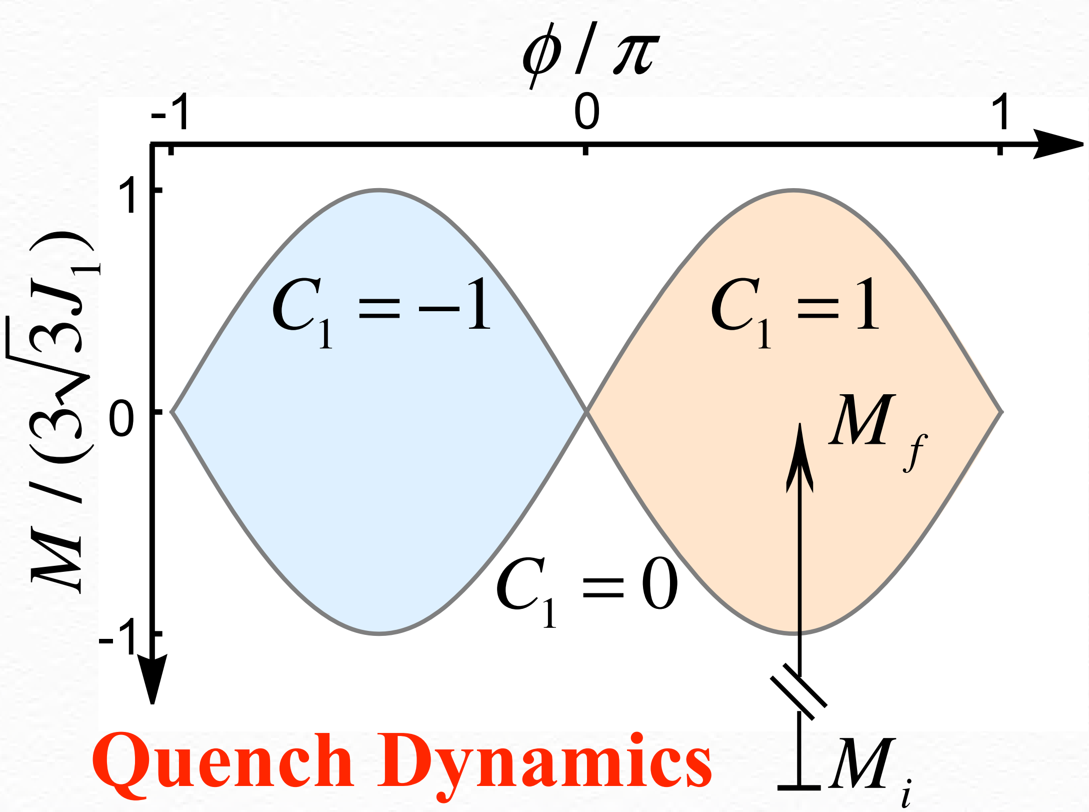
Quantized Edge State
Quantized Hall Conductance

?

Bulk-Edge Correspondence



Xue's group
Science 2013

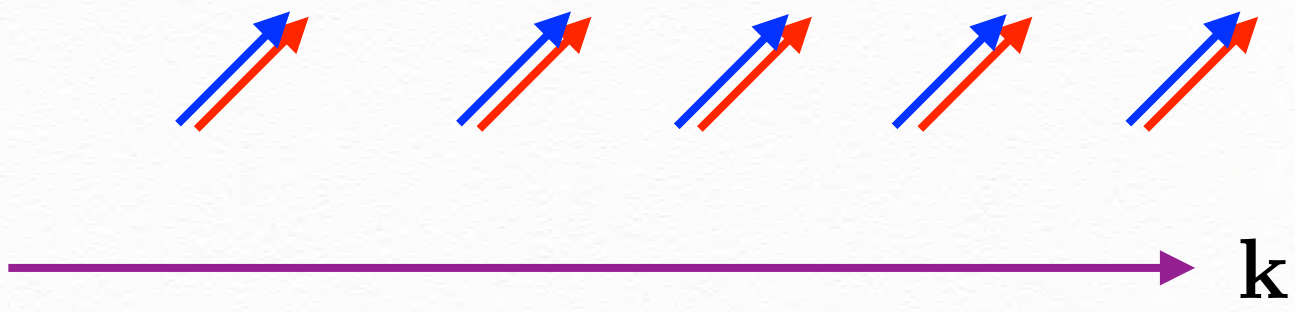


Quench Dynamics

Description of Quench Dynamics

A two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

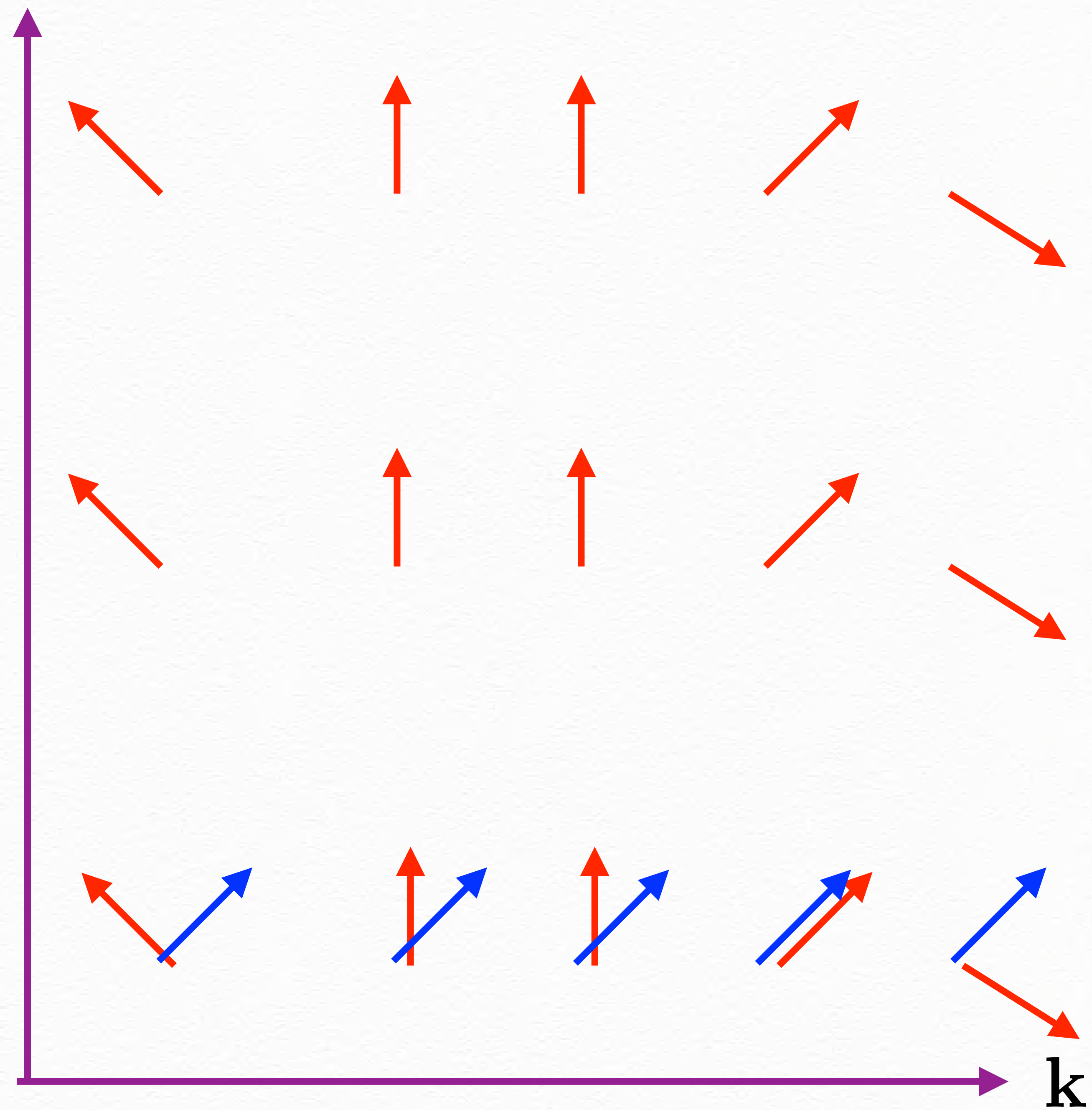
Initial hamiltonian $\mathbf{h}^{\mathrm{i}}(\mathbf{k})$



Description of Quench Dynamics

A two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

Quench from $\mathbf{h}^{\mathrm{i}}(\mathbf{k})$ ➡ $\mathbf{h}^{\mathrm{f}}(\mathbf{k})$



Description of Quench Dynamics

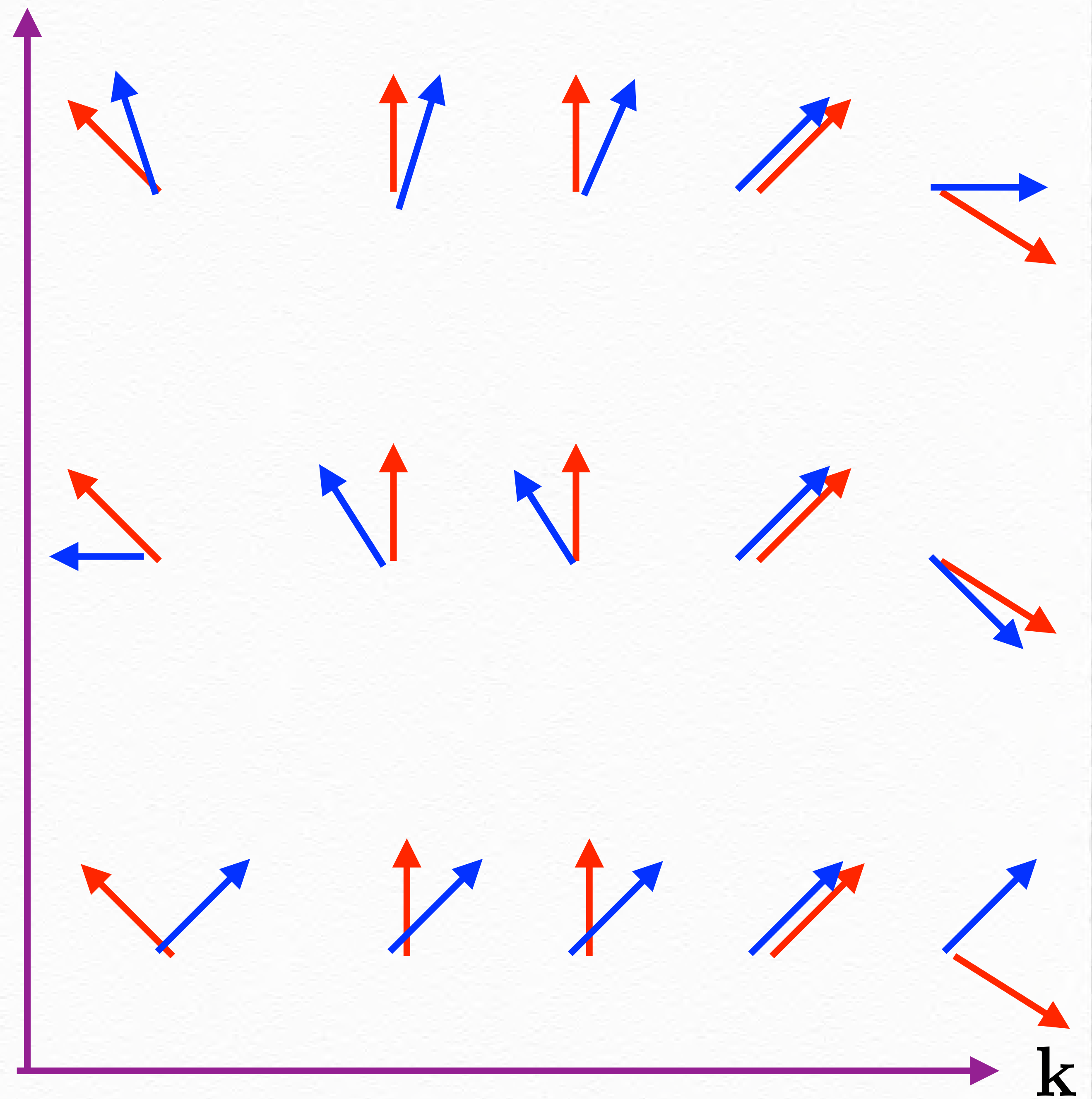
A two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

Quench from $\mathbf{h}^{\mathrm{i}}(\mathbf{k}) \Rightarrow \mathbf{h}^{\mathrm{f}}(\mathbf{k})$

$$\zeta(\mathbf{k}, t) = \exp\left\{-\frac{i}{2}\mathbf{h}^{\mathrm{f}}(\mathbf{k}) \cdot \boldsymbol{\sigma} t\right\} \zeta^{\mathrm{i}}(\mathbf{k}),$$

$$\mathbf{n} = \zeta^{\dagger}(\mathbf{k}, t)\boldsymbol{\sigma}\zeta(\mathbf{k}, t),$$

$$[k_x, k_y, t] \Rightarrow \mathbf{n}$$



Theorem: Topology from Dynamics

For a two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

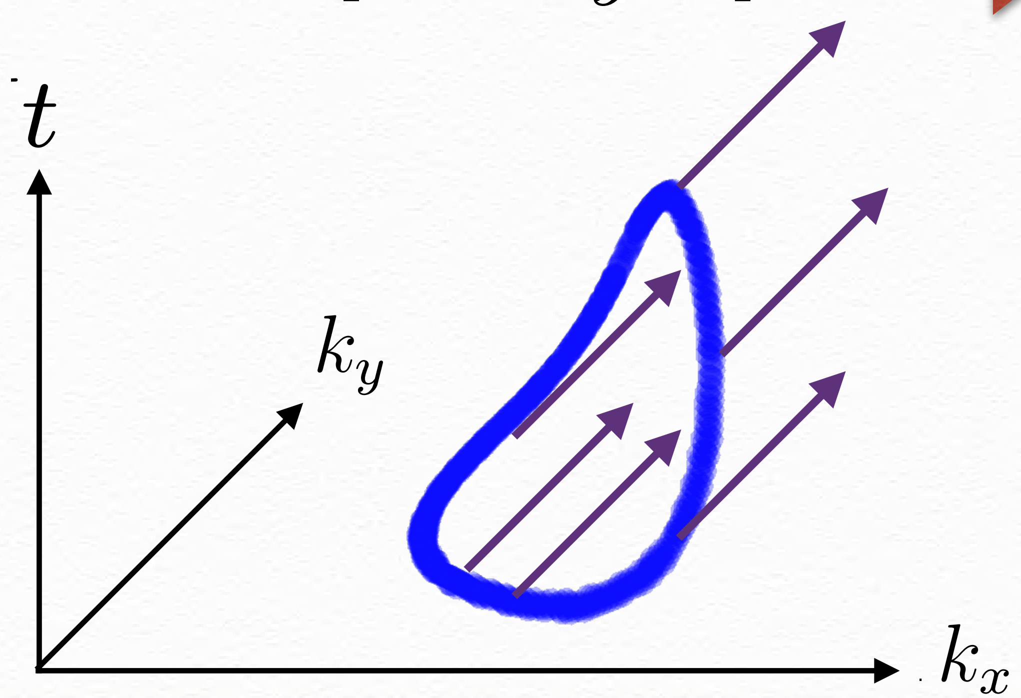
Considering the quench dynamics described by:

$$\zeta(\mathbf{k}, t) = \exp\left\{-\frac{i}{2}\mathbf{h}^{\mathrm{f}}(\mathbf{k}) \cdot \boldsymbol{\sigma} t\right\} \zeta^{\mathrm{i}}(\mathbf{k}),$$

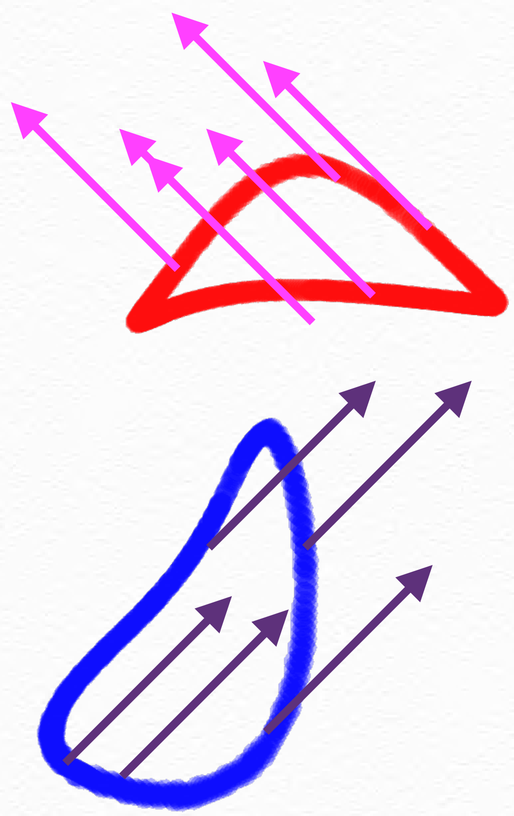
$$\mathbf{n} = \zeta^{\dagger}(\mathbf{k}, t)\boldsymbol{\sigma}\zeta(\mathbf{k}, t),$$

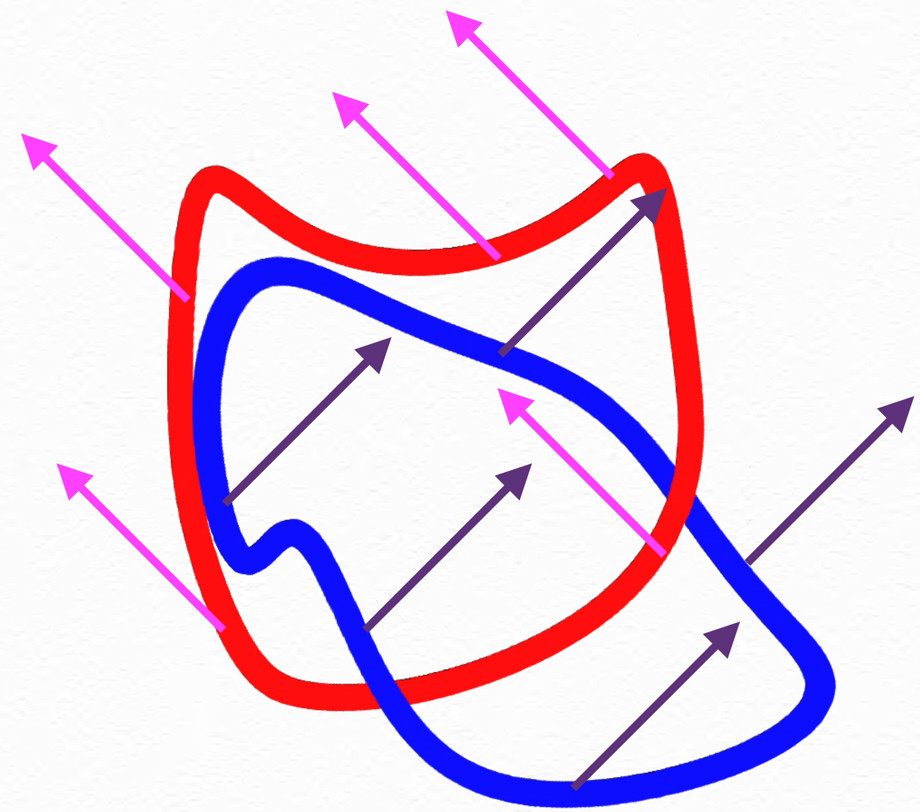
this defines a Hopf map f **:** $[k_x, k_y, t] \longrightarrow \mathbf{n}$

$f^{-1}(\mathbf{n}_1)$



Theorem: Topology from Dynamics





linking number = 0

linking number = 1

The linking number of $f^{-1}(\mathbf{n}_1)$ and $f^{-1}(\mathbf{n}_2)$

= The Chern number of the final Hamiltonian

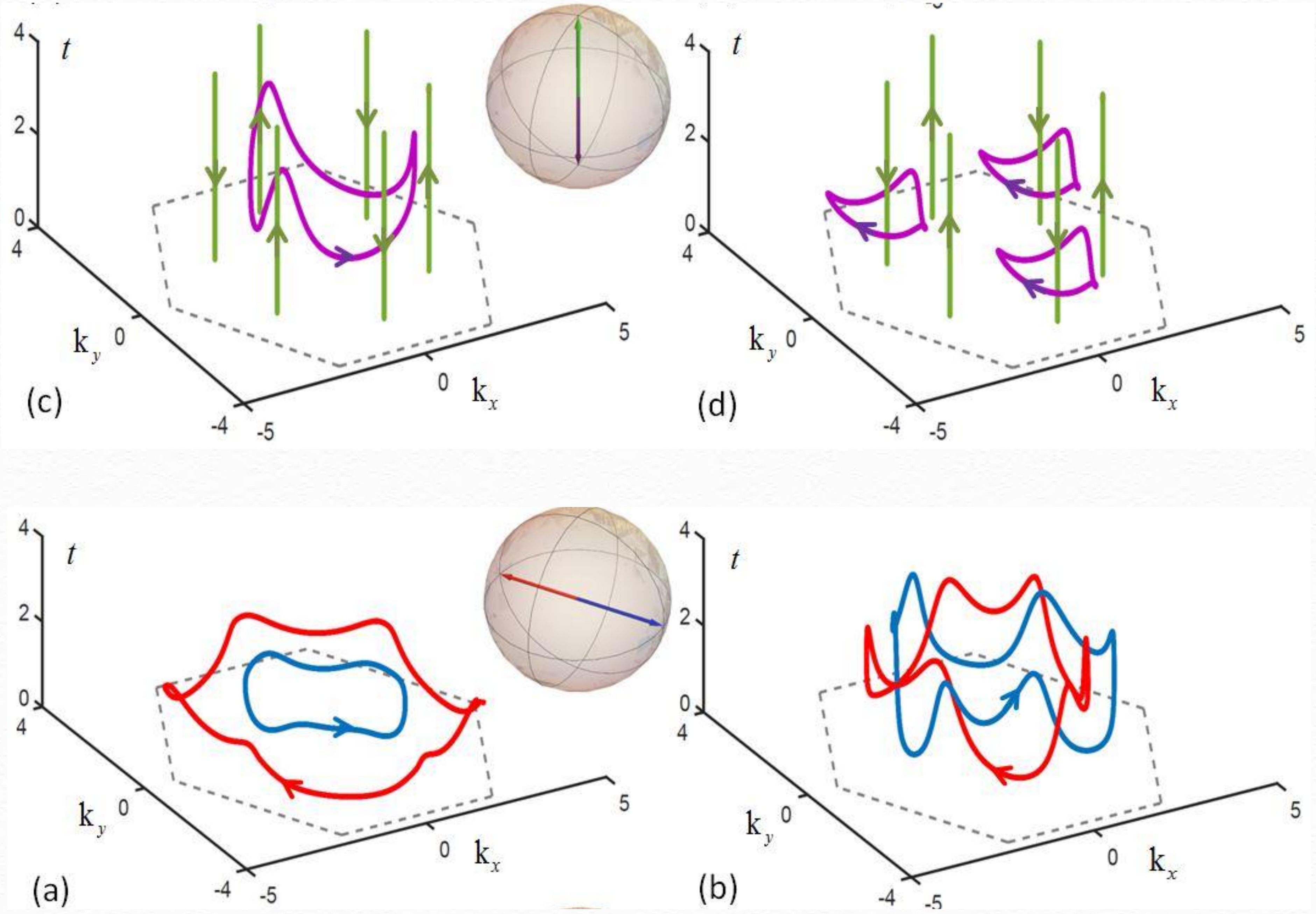
$$\Pi_3(S^2) = \Pi_2(S^2) = Z$$

Ce Wang, Pengfei Zhang, Xin Chen, Jinlong Yu and Hui Zhai, PRL (2017)

Example of Theorem

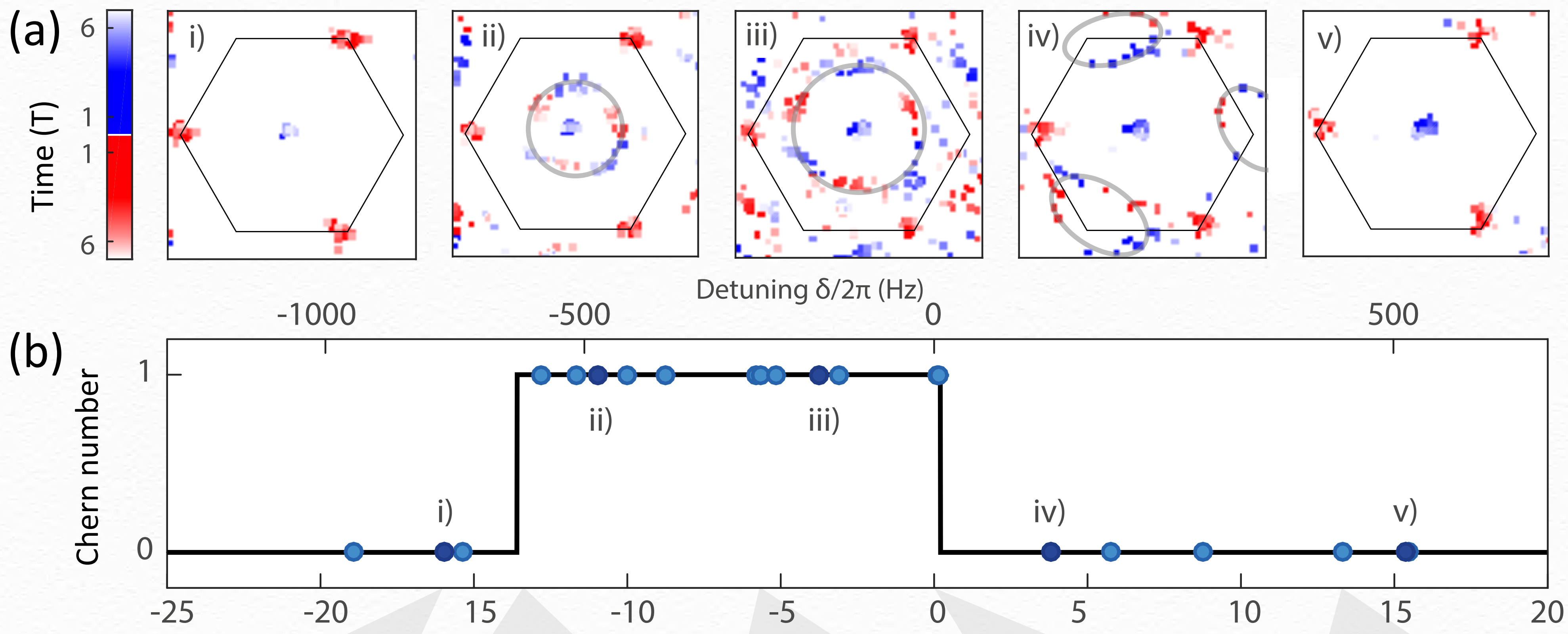
Topological Trivial

Topological Non-trivial



Experimental Observations

Haldane Model: Hamburg group



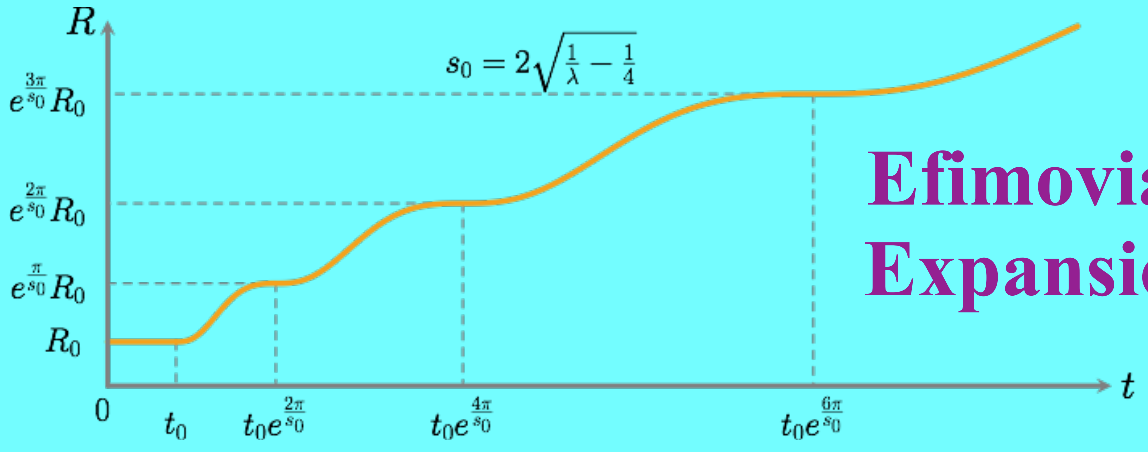
Nat. Comm. 2019

See similar result from USTC group

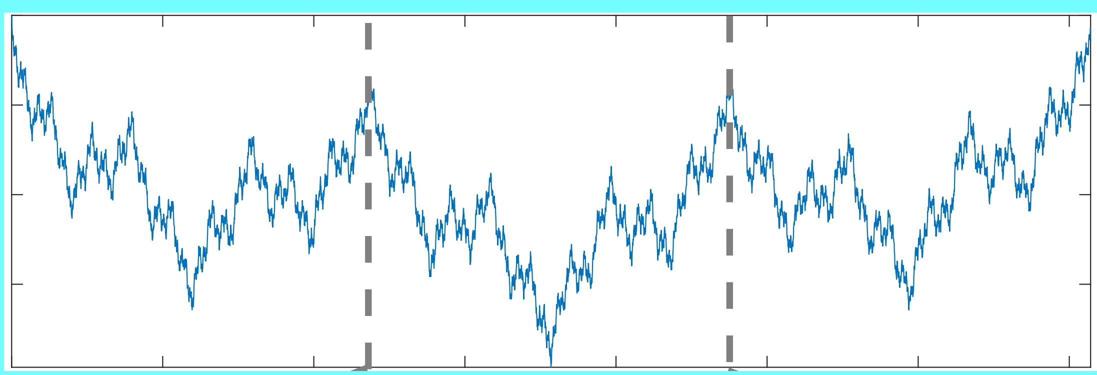
We thereby map out the trivial and non-trivial Chern number areas of the phase diagram. As shown by Wang et al. (ref. [13]), the Chern number of the post quench Hamiltonian maps onto the linking number between this contour and the position of the static vortices [Fig. 1(a)]. We thus demonstrate that the direct mapping between two topological indices – a static and a dynamical one – allows for an unambiguous measurement of the Chern number.

Take-Home Message

Symmetry and Topology can be detected from non-equilibrium dynamics.



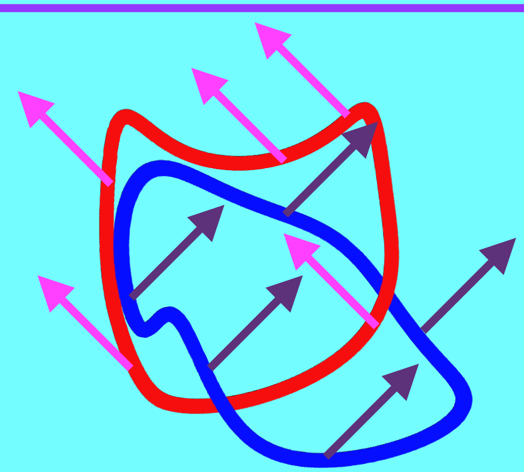
Efimovian Expansion



Dynamical Fractal

Symmetry

Linking Number



Topology

Thank You Very Much for Attention !

Fundamental Problems in Quantum Non-Equilibrium Dynamics II

Hui Zhai

Institute for Advanced Study
Tsinghua University

CSRC Workshop on Quantum Non-Equilibrium Phenomena
June 2019

What this is all about ?

Hayden and Preskill ask:

Can one retrieval information from a black hole ?

What this is all about ?

Hayden and Preskill ask:

Can one retrieval information from a black hole ?

Why you talk about this HERE?

Introduction

- Quantum Thermalization
- Out-of-Time-Ordered Correlation
- Thermofield Double State

Introduction

- Quantum Thermalization
- Out-of-Time-Ordered Correlation
- Thermofield Double State

Quantum Thermalization

$|\Psi\rangle$

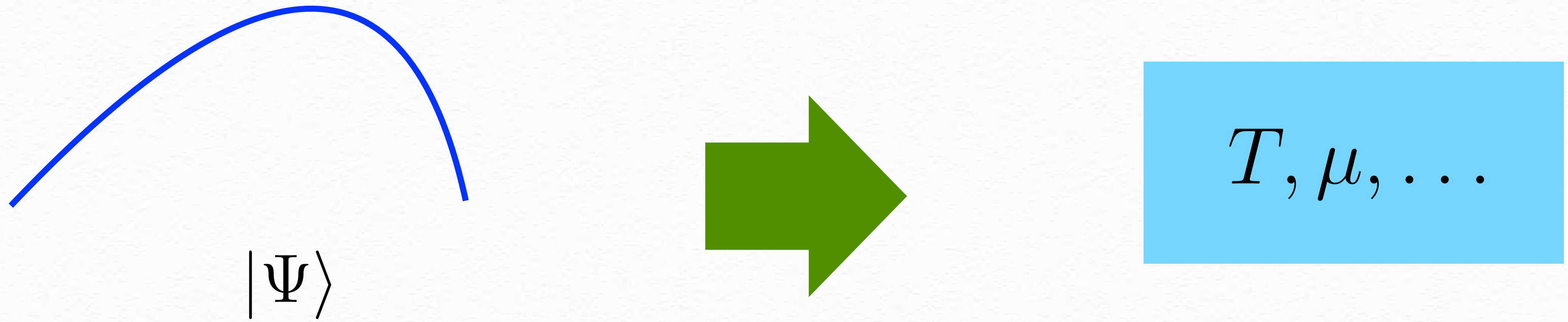
$$T, \mu, \ldots$$

Quantum wave function

Unitary evolution

Thermal equilibrium

Eigenstate Thermalization Hypothesis



Quantum wave function — **Unitary evolution** — **Thermal equilibrium**

Eigenstate Thermalization Hypothesis

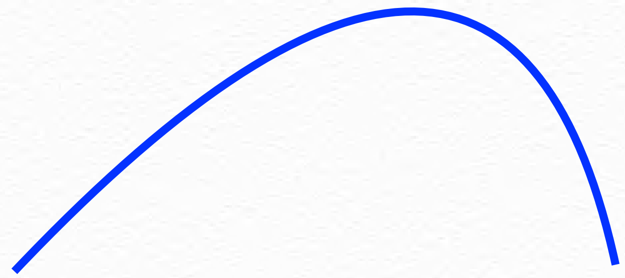
$$\langle \hat{\mathcal{O}} \rangle_\infty = \langle \rho_{eq}(E) \hat{\mathcal{O}} \rangle$$

Local observable

Equilibrium Density Matrix of the Whole System

Sufficient Long Time Evolution

Quantum Thermalization “ Paradox ”



$|\Psi\rangle$

$T, \mu, \ldots$

Quantum wave function	Unitary evolution	Thermal equilibrium
Contains local information	**Perserve Information**	**Where is the information?**

Paradox:

Information Scrambling

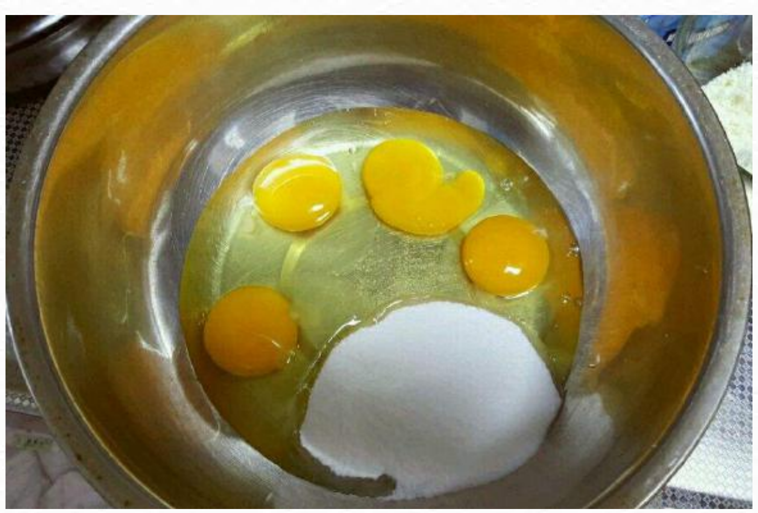
$|\Psi\rangle$

$T, \mu, \ldots$

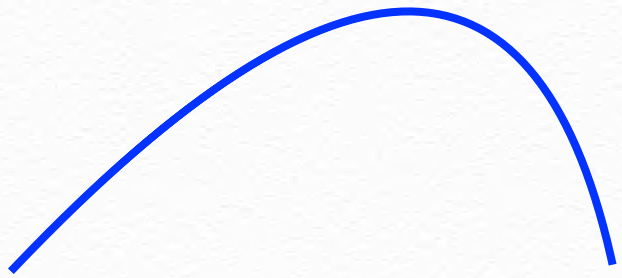
Quantum wave function

Unitary evolution

Thermal equilibrium



Black Hole Information Paradox



$|\Psi\rangle$

$T, \mu, \ldots$

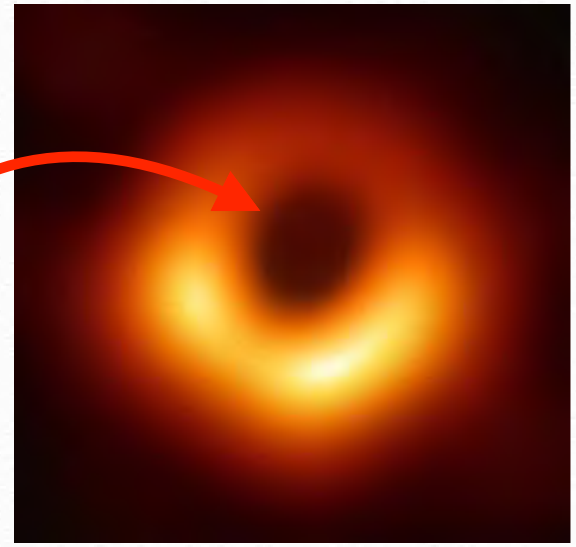
Quantum wave function

Unitary evolution

Thermal equilibrium

Alice

$|\Psi\rangle$



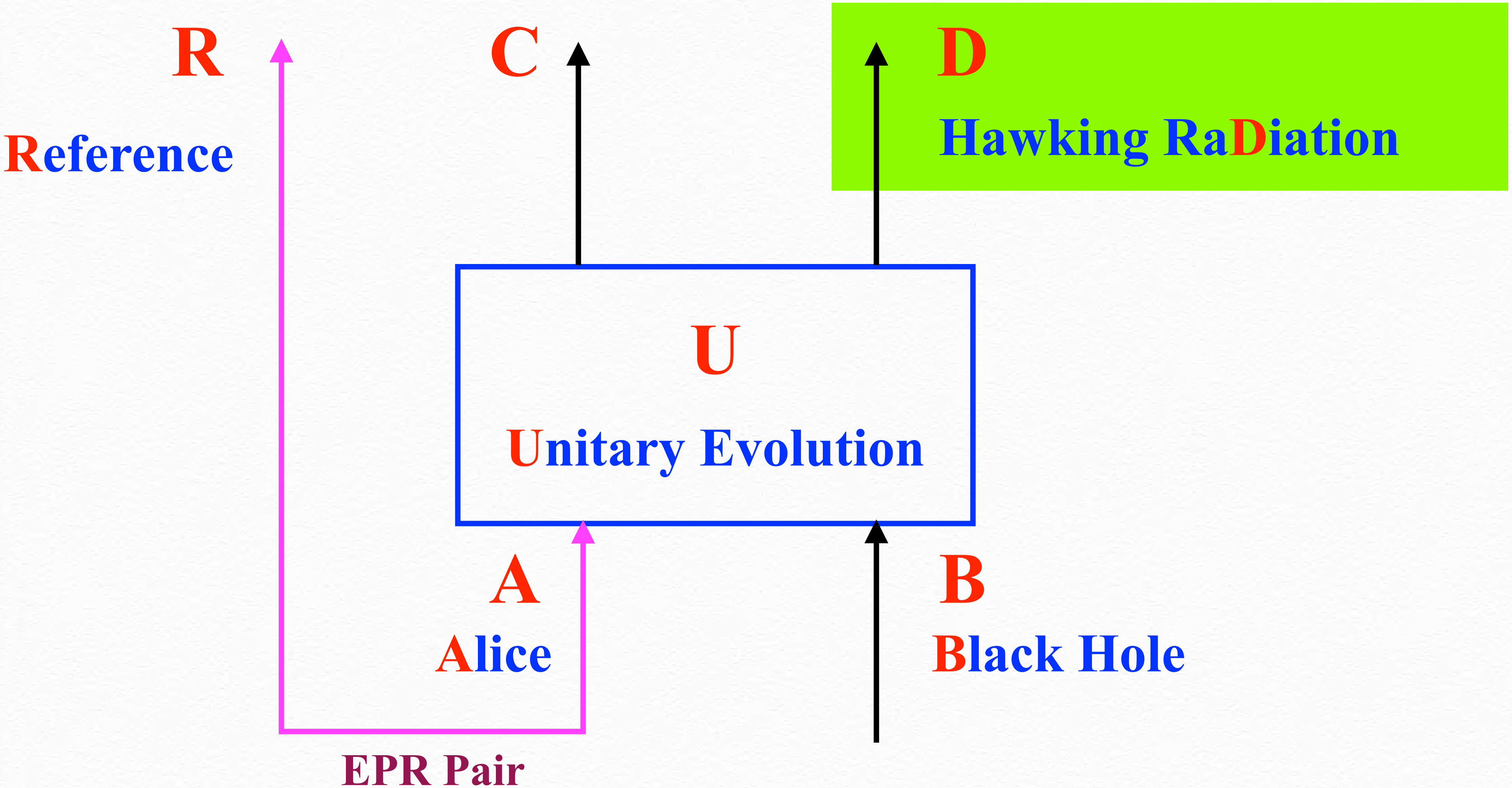
+ Hawking Radiation

Blackhole has no hair

Where is the information?

Quantum Information Perspective

$$\mathcal{H}_B \gg \mathcal{H}_D \gg \mathcal{H}_A$$

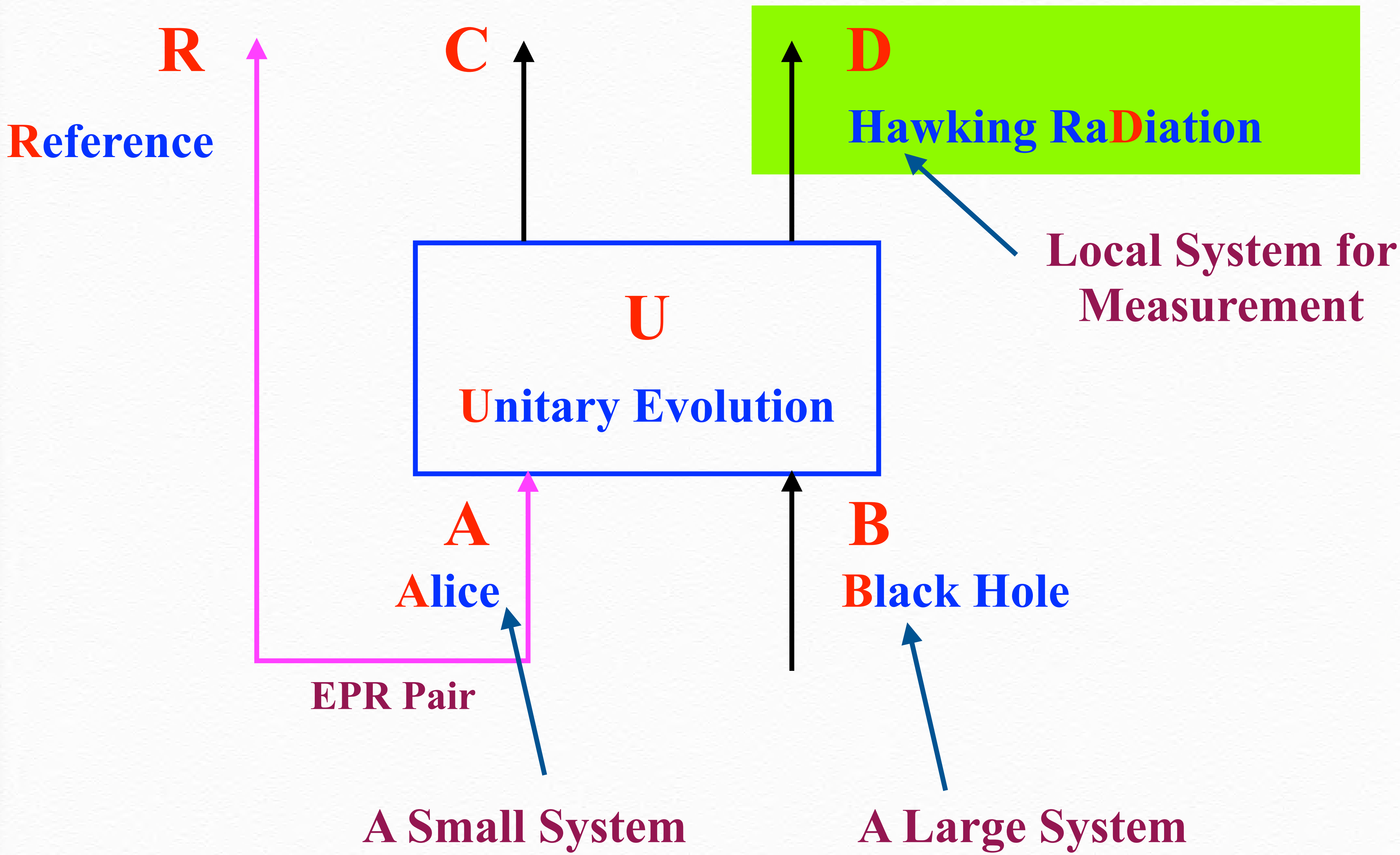


$$I^{(2)}(R, D) = 0$$

Page 1993, Hayden, Priskill, 2007

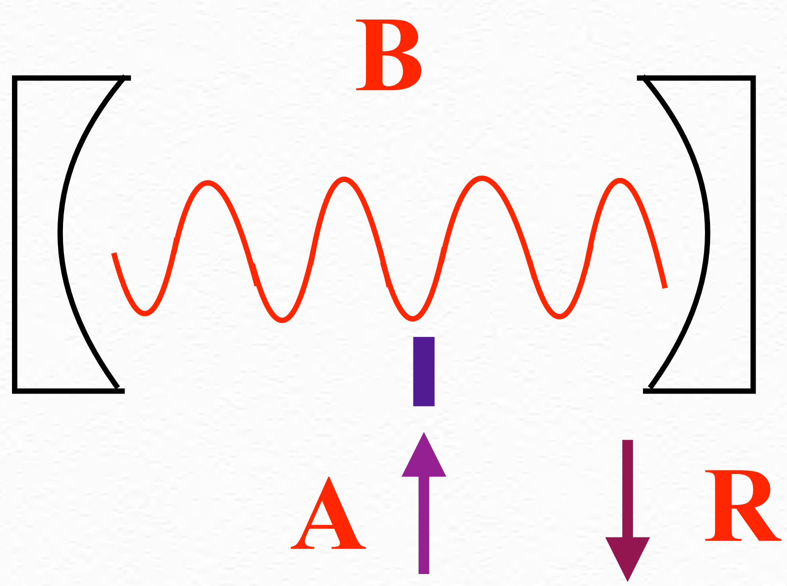
Quantum Information Perspective

$$\mathcal{H}_B \gg \mathcal{H}_D \gg \mathcal{H}_A$$



Dicke Model Realization

$$\hat{H} = \hbar\omega_0 a^\dagger a + \qquad\qquad + \omega_z \sigma_z$$



A Large System

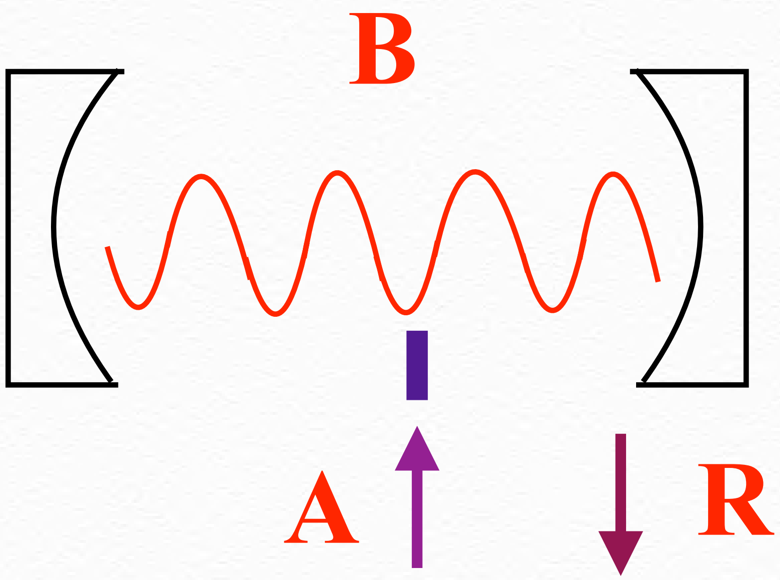
$$\{|n\rangle\}$$

$$|\Psi\rangle_{AR} = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$$

$$\rho^{\mathrm{i}}_{\mathrm{RA}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Dicke Model Realization

$$\hat{H} = \hbar\omega_0 a^\dagger a + g(a^\dagger + a)\sigma_x + \omega_z\sigma_z$$



A Large System

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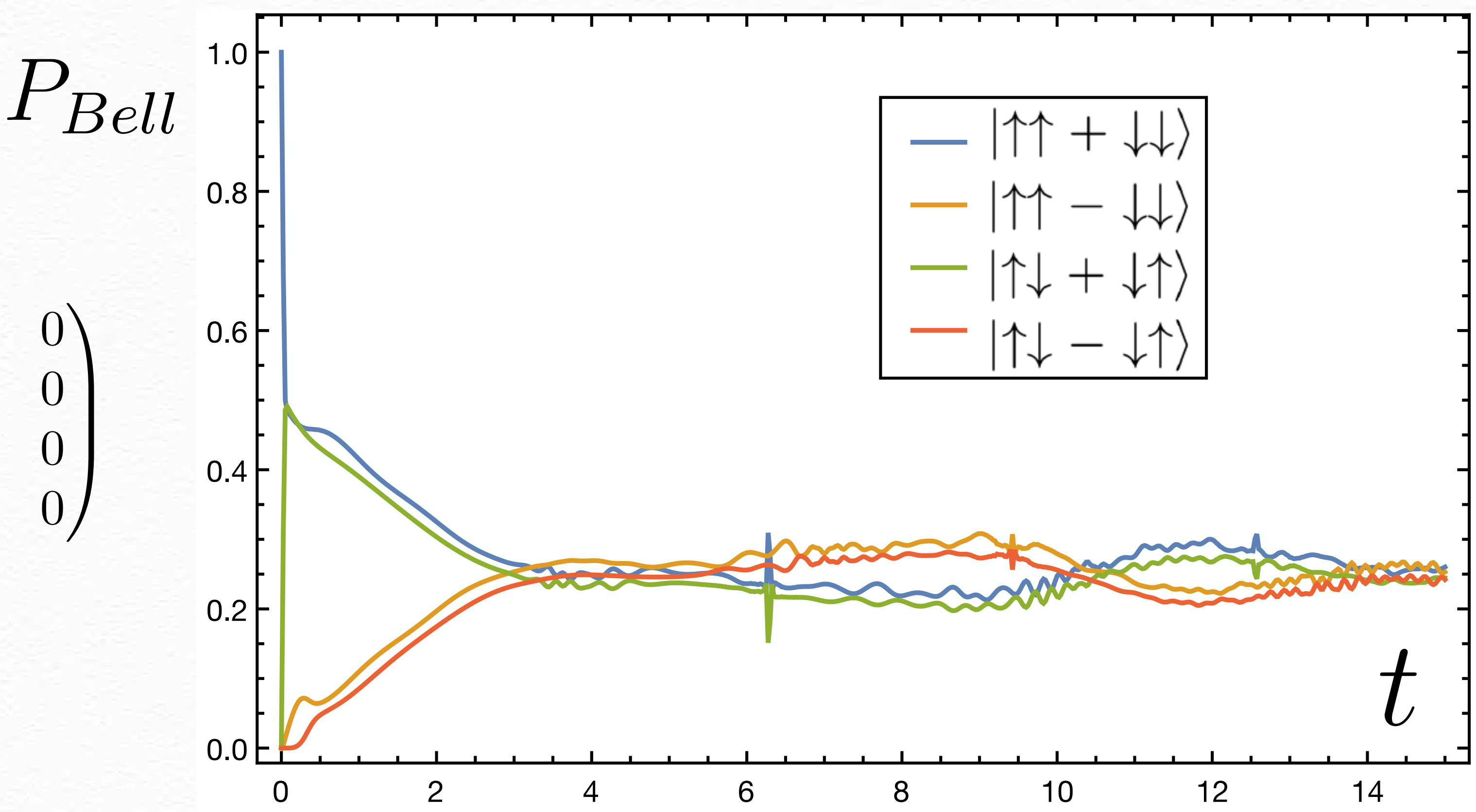


$$\rho^{\mathrm{f}}_{\mathrm{RA}} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Introduction

- Quantum Thermalization
- Out-of-Time-Ordered Correlation
- Thermofield Double State

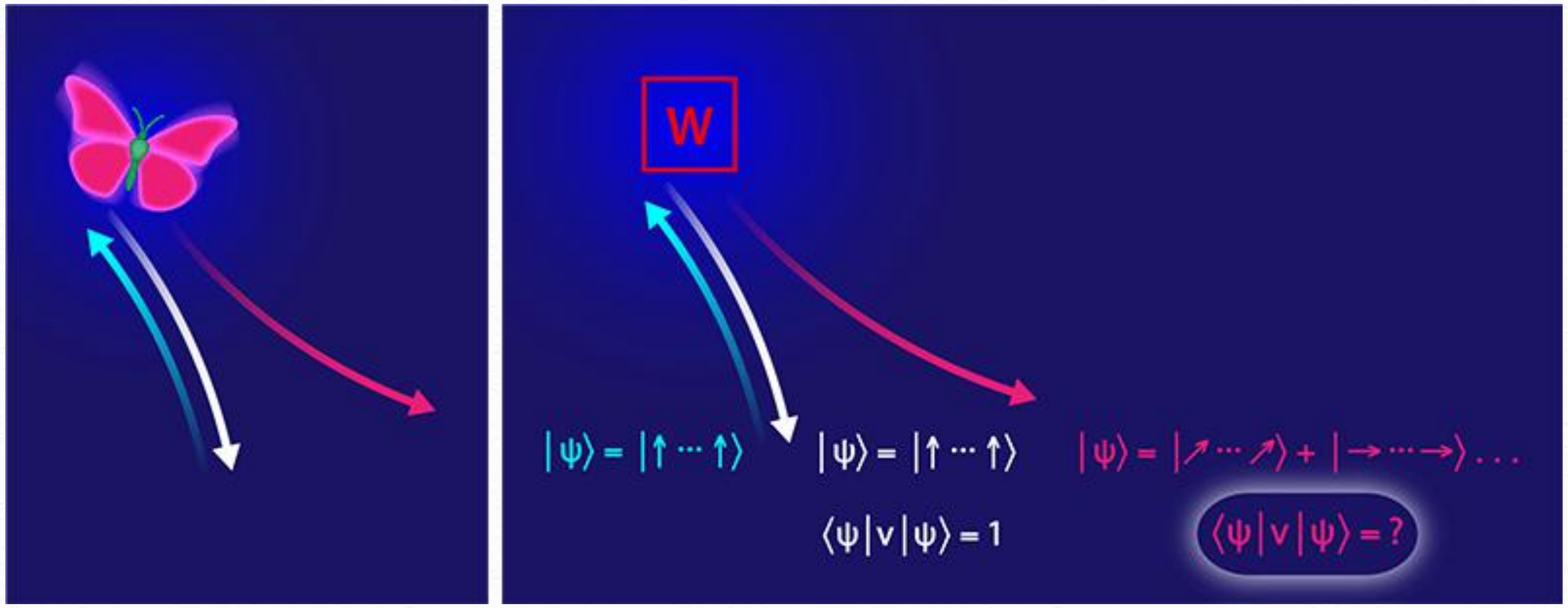
Out-of-time-ordered Correlation and Chaos

$$\langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle_\beta$$

$$\hat{W}(t) = e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}$$

- **OTOC measures the difference when exchanging orders of two operations**

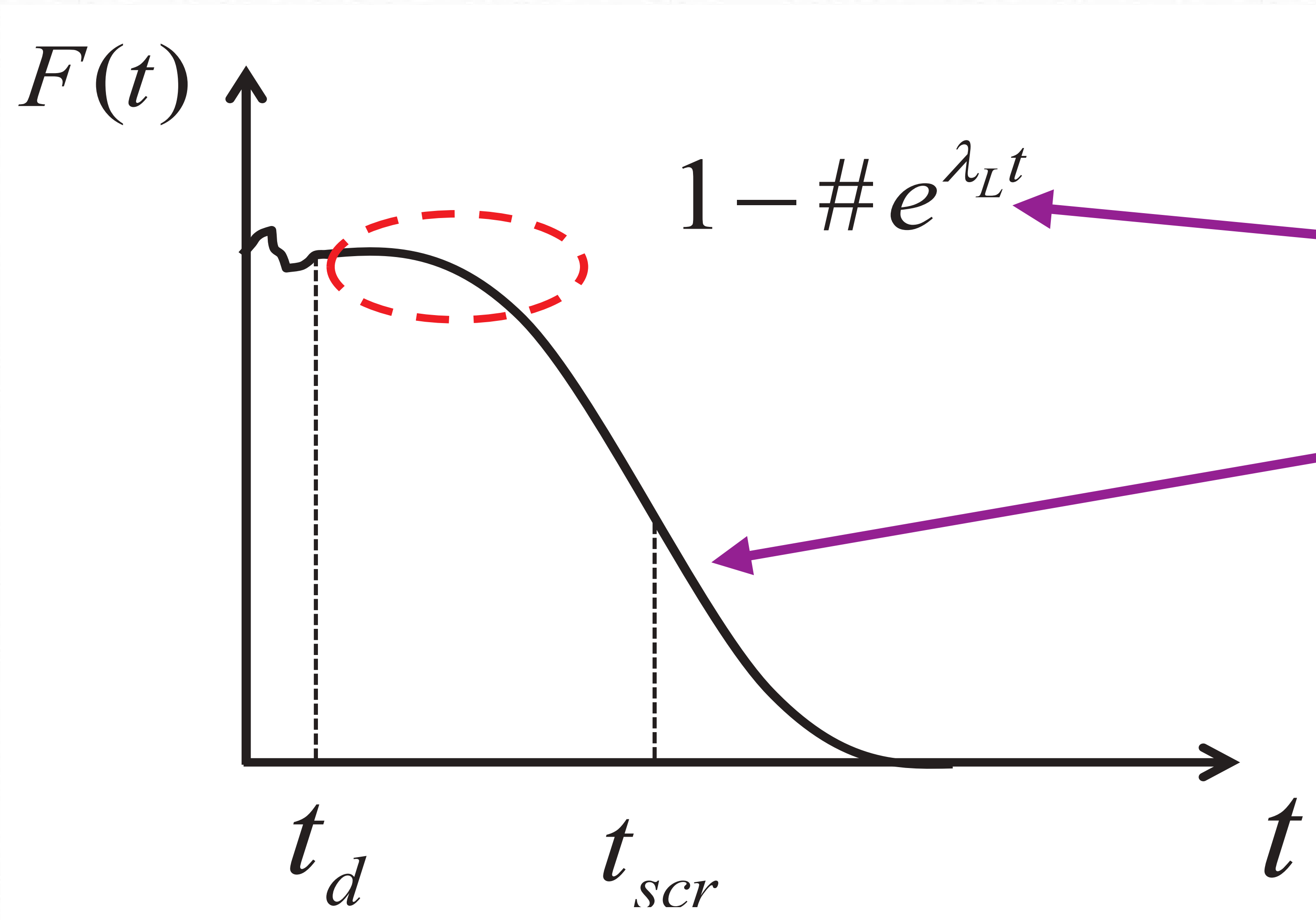
$$\hat{W}(t)\hat{V}(0)|\rangle \qquad \hat{V}(0)\hat{W}(t)|\rangle$$



Out-of-time-ordered Correlation

$$\langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle_\beta$$

$$\hat{W}(t) = e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}$$

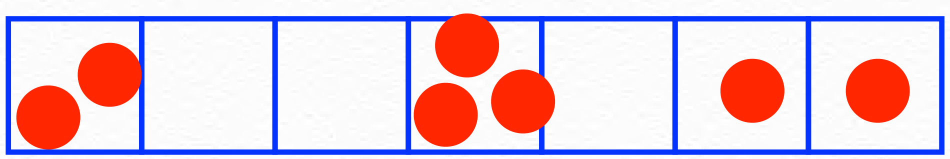


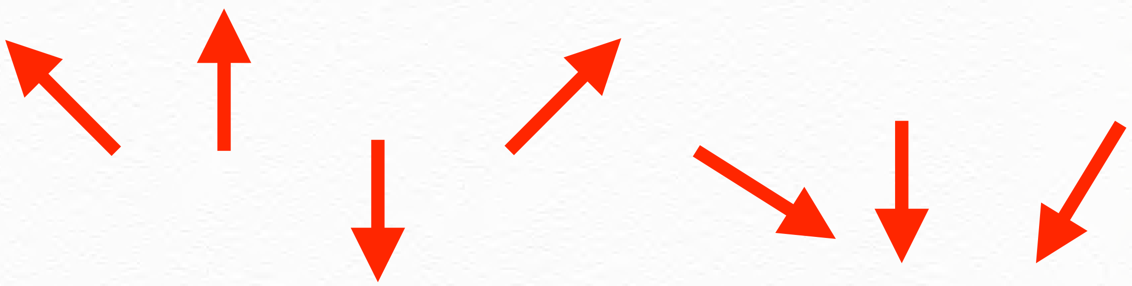
Lyapunov exponent

Scrambling time

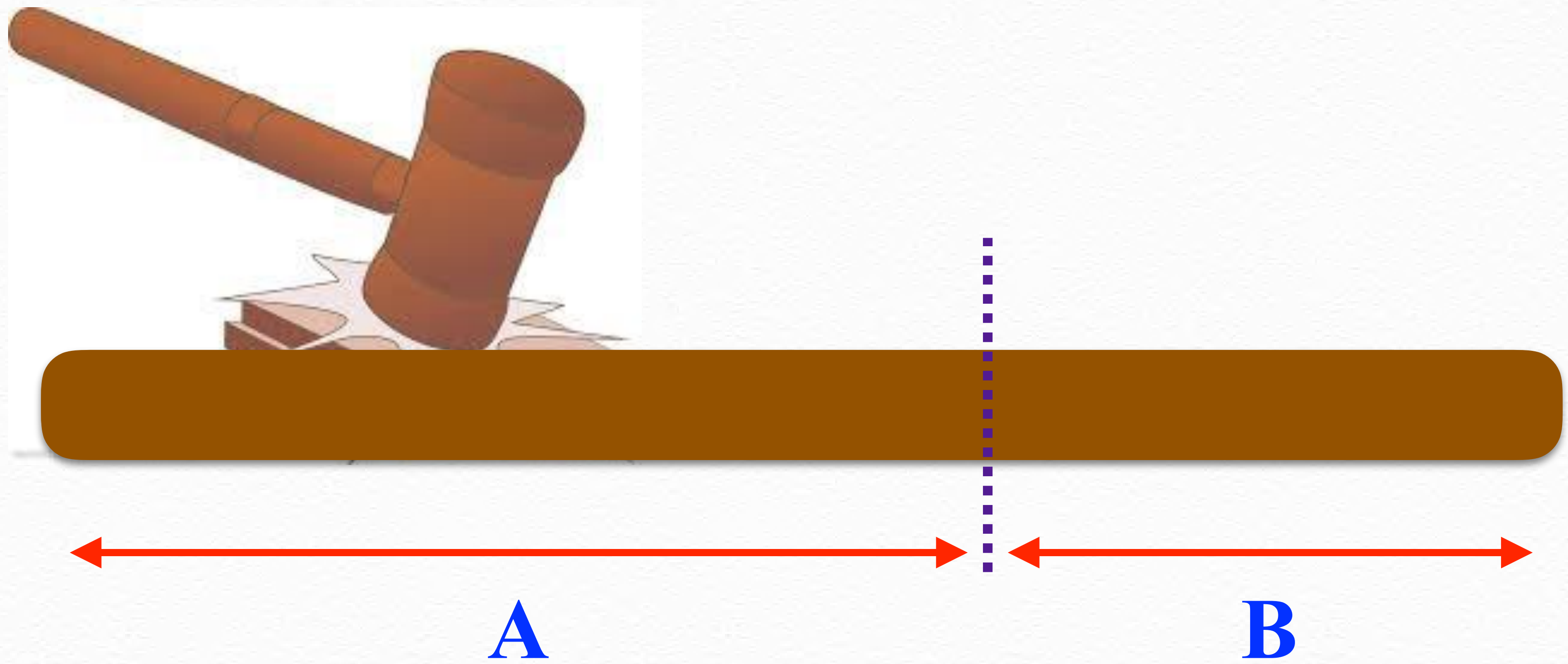
Quench Experiment

Local quench $\hat{b}_i^\dagger|\Psi\rangle$ $S_i^-|\Psi\rangle$





Quench Experiment



$$\rho_A = \mathrm{Tr}_B \rho$$

The Second Renyi Entropy $$S_A^{(2)} = -\log \mathrm{Tr}_A \hat{\rho}_A^2$$

OTOC: Information Scrambling

$$\exp(-S_A^{(2)}) = \sum_{M\in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$

Non-Equilibrium Properties

Quench the system by arbitrary operator $\mathcal{O}$

Entanglement Entropy

=

Equilibrium Properties

$$\hat{V} = \hat{O}\hat{O}^{\dagger}$$

$\hat{M}$ **is a complete set of operators in B**

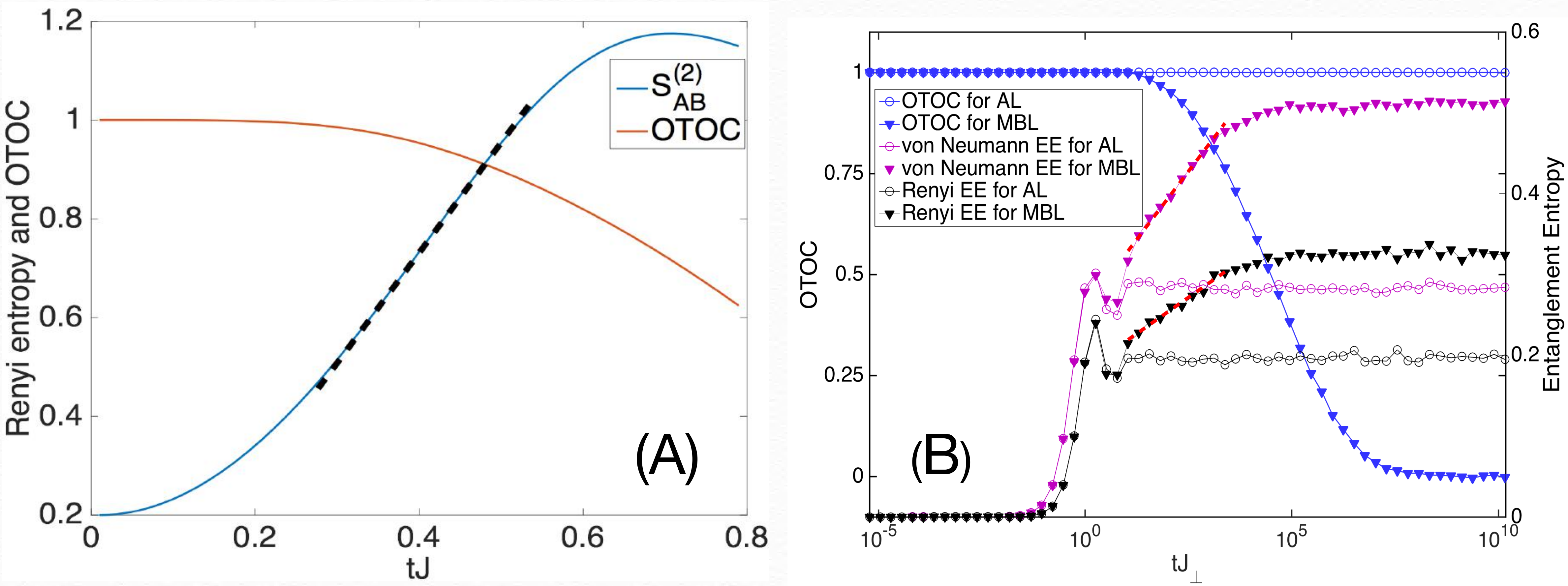
OTOC

Highly Cited Paper

Fan, Zhang, Shen, Zhai, Science Bulletin, 2017

OTOC: Information Scrambling

$$\exp(-S_A^{(2)}) = \sum_{M\in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$



Thermal Phase (ETH):
Bose-Hubbard Model

Single-Particle Localized and MBL:
XXZ Model + Random field

Shen, Zhang, Fan, Zhai, PRB, 2017
Fan, Zhang, Shen, Zhai, Science Bulletin, 2017

OTOC: Information Scrambling

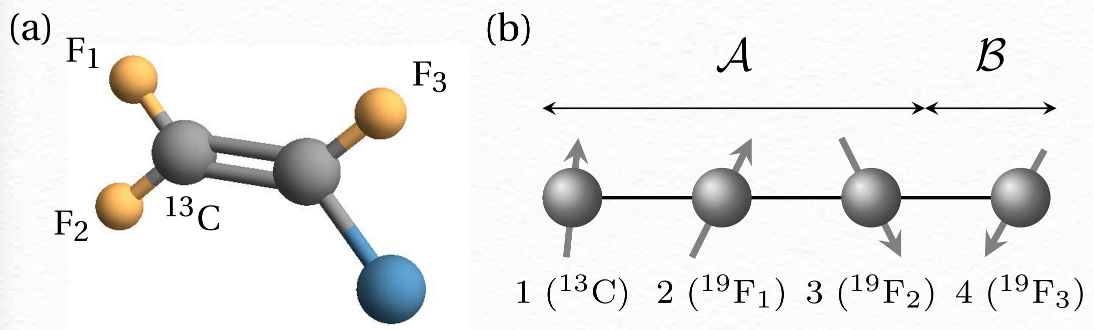
$$\exp(-S_A^{(2)}) = \sum_{M\in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$

Thermal Phase (ETH)	Single-Particle Localized	Many-Body Localized
Linear increasing of entanglement	No spreading of entanglement	Logarithmic spreading of entanglement
OTOC exponential decay	OTOC remains constant	OTOC power-law decay

Our Results

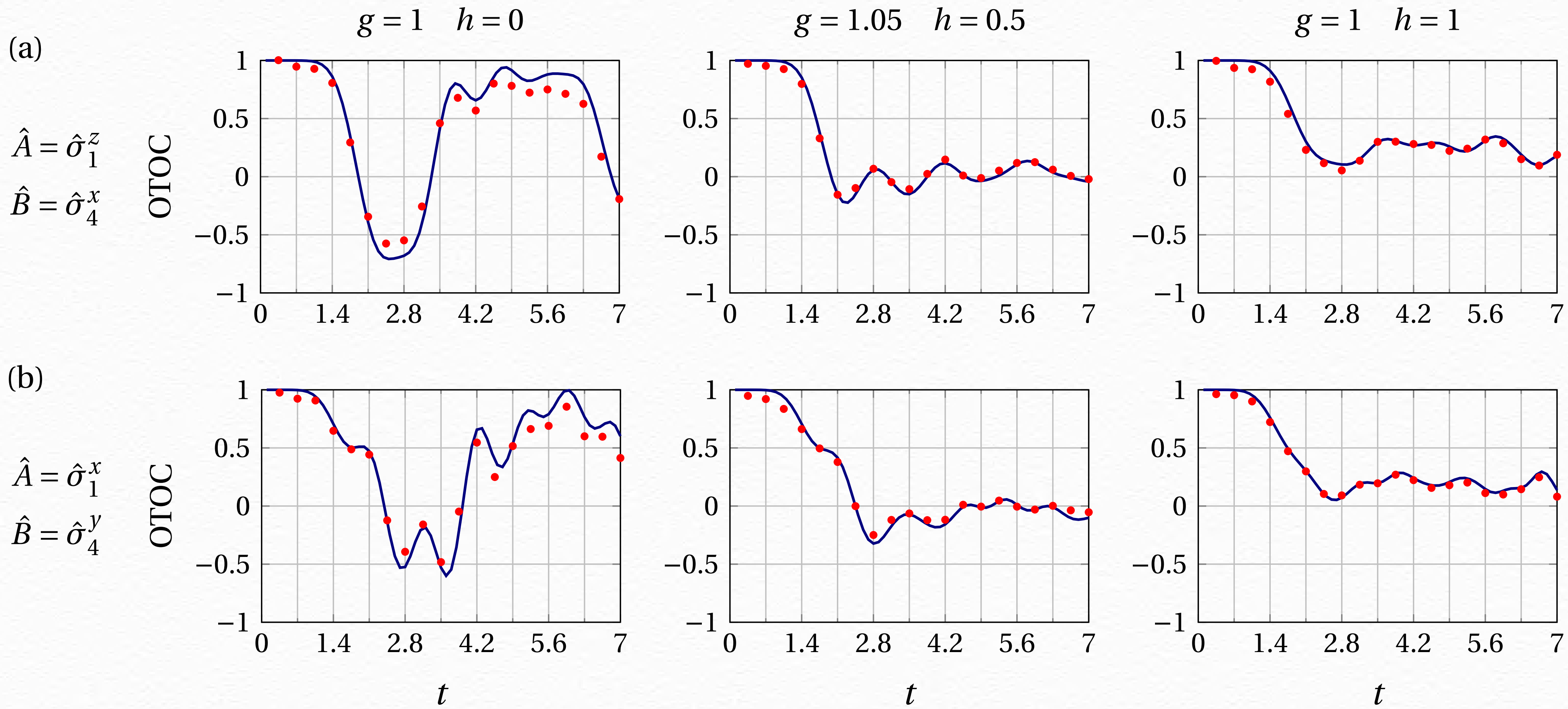
Fan, Zhang, Shen, Zhai, Science Bulletin, 2017

Measurements of OTOC for Ising Chain



$$\hat{H} = \sum_i \left(-\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + g\hat{\sigma}_i^x + h\hat{\sigma}_i^z\right)$$

Integrable Case **Non-Integrable Cases**

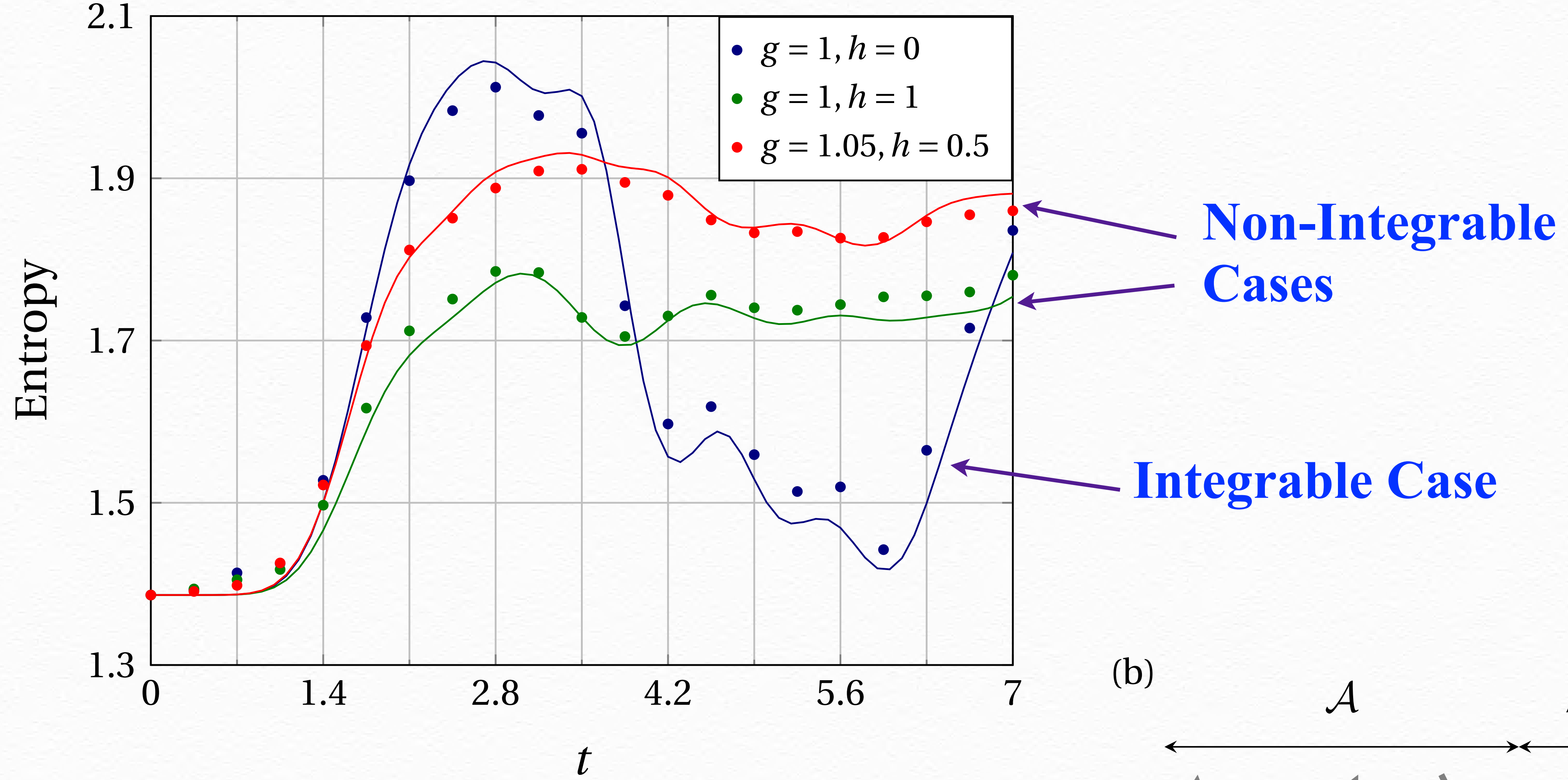


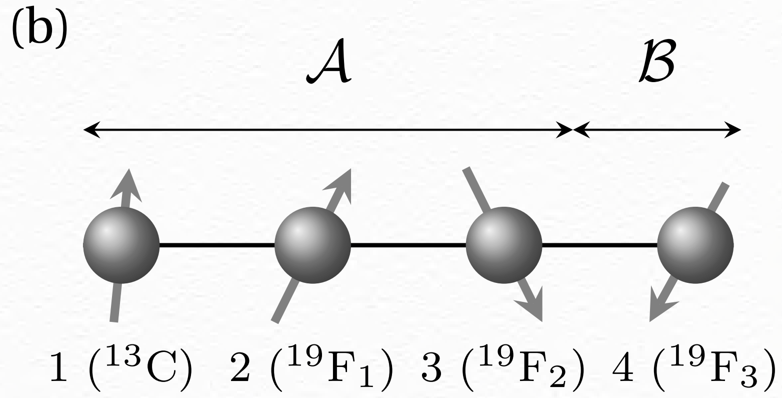
Jun Li et.al. PRX 2017

See also,
M. Garttner, et.al. Nat. Phys. 2017

Measurements of OTOC for Ising Chain

$$\exp(-S_A^{(2)}) = \sum_{M \in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$





Integrable Case: Information Oscillates

Non-Integrable Case: Information Scrambles

Jun Li et.al. PRX 2017

OTOC: Holographic Duality

Quantum Side

- Lyapunov exponent has a upper bound

$$\lambda_L \leqslant \frac{2\pi}{\beta}$$

Gravity Side

- OTOC has also emerged, and with a black hole

$$\lambda_L = \frac{2\pi}{\beta}$$

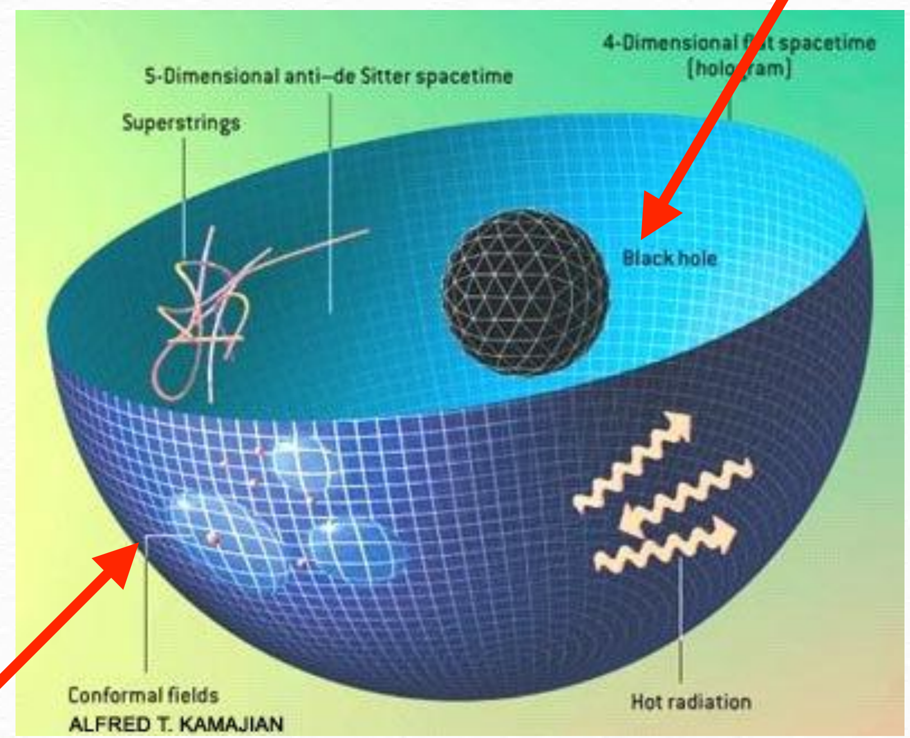
A quantum system with holographically dual to a black hole saturates the bound
== Black hole is a faster scrambler in nature

An example is the SYK model

Kitaev, KITP, 2015; Maldacena, Shenker and Stanford, 2015

OTOC: Holographic Duality

A gravity theory in D+1-dimension

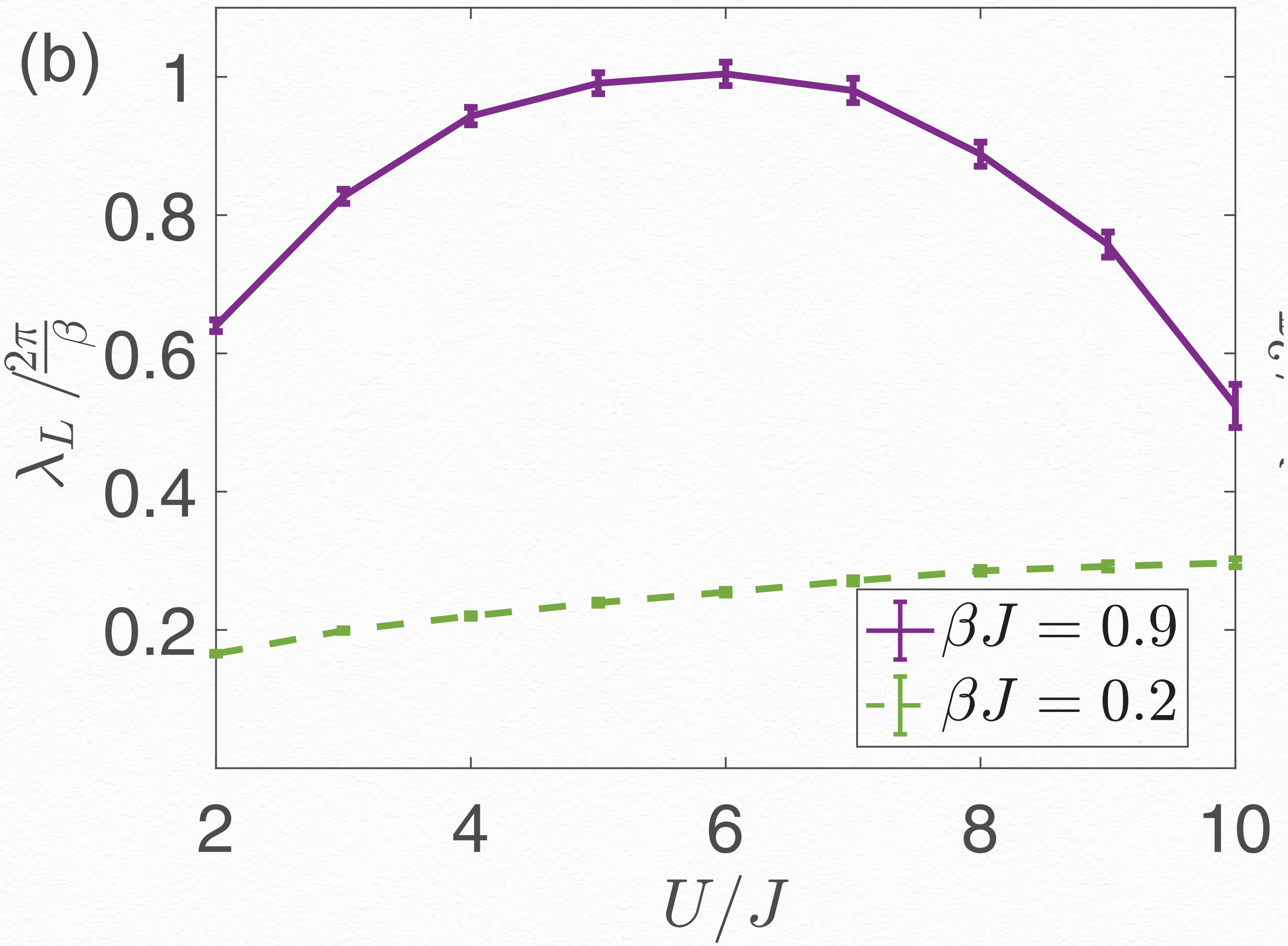


A quantum many-body system in D-dimension (strongly interacting, emergent conformal field symmetry)

Kitaev, KITP, 2015; Maldacena, Shenker and Stanford, 2015

OTOC for Bose-Hubbard Model

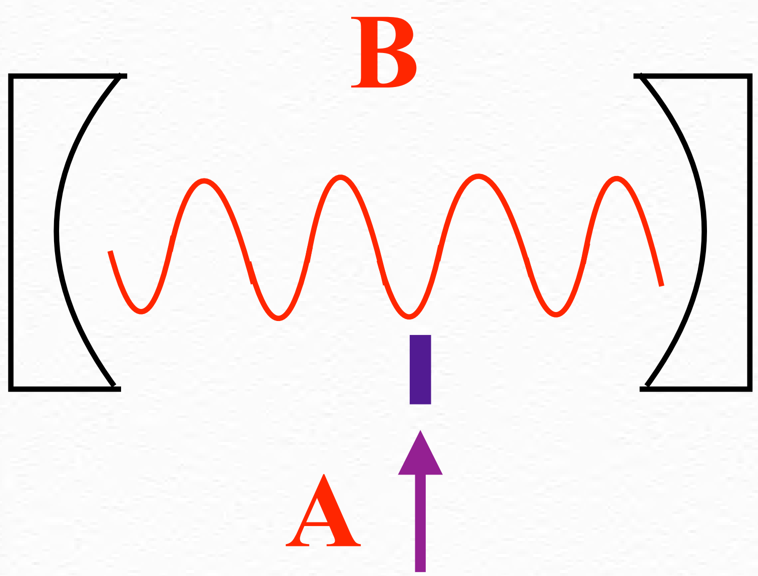
$$\hat{H} = -J \sum_{\langle ij \rangle} (\hat{b}_i^\dagger \hat{b}_j + \text{H.c.}) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

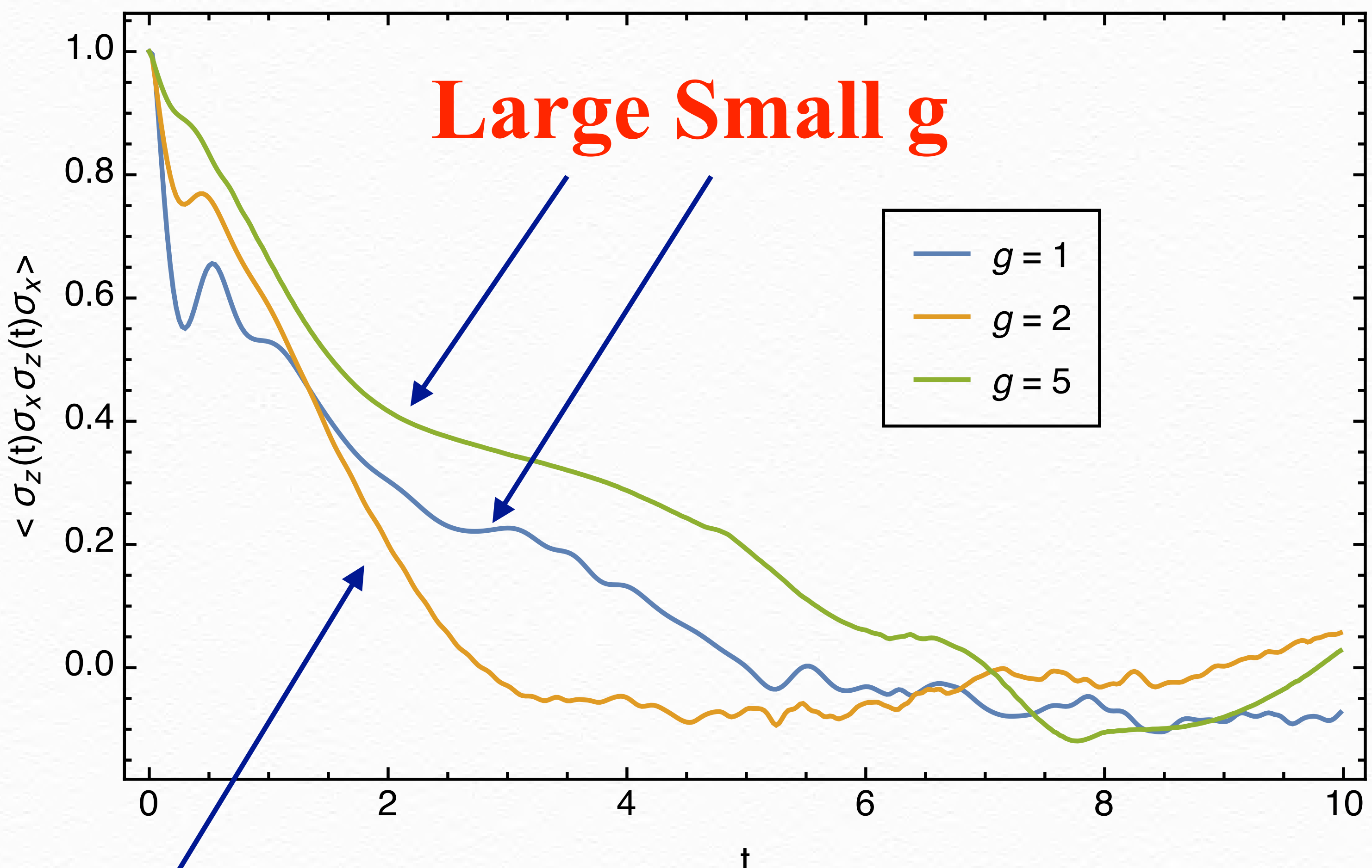


Shen, Zhang, Fan, Zhai, PRB, 2017

OTOC for Dicke Model

$$\hat{H} = \hbar\omega_0 a^\dagger a + g(a^\dagger + a)\sigma_x + \omega_z\sigma_z$$





Intermediate g

Introduction

- Quantum Thermalization
- Out-of-Time-Ordered Correlation
- Thermofield Double State

Thermofield Double State

Left

$$\{|n\rangle_L\}$$

Right

$$\{|n\rangle_R\}$$

$$|\Psi\rangle_{TFD} = \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R$$

- $$\mathrm{Tr}_R |\Psi\rangle\langle\Psi| = \sum_n e^{-\beta E_n} |n\rangle\langle n|$$

Generalized EPR State

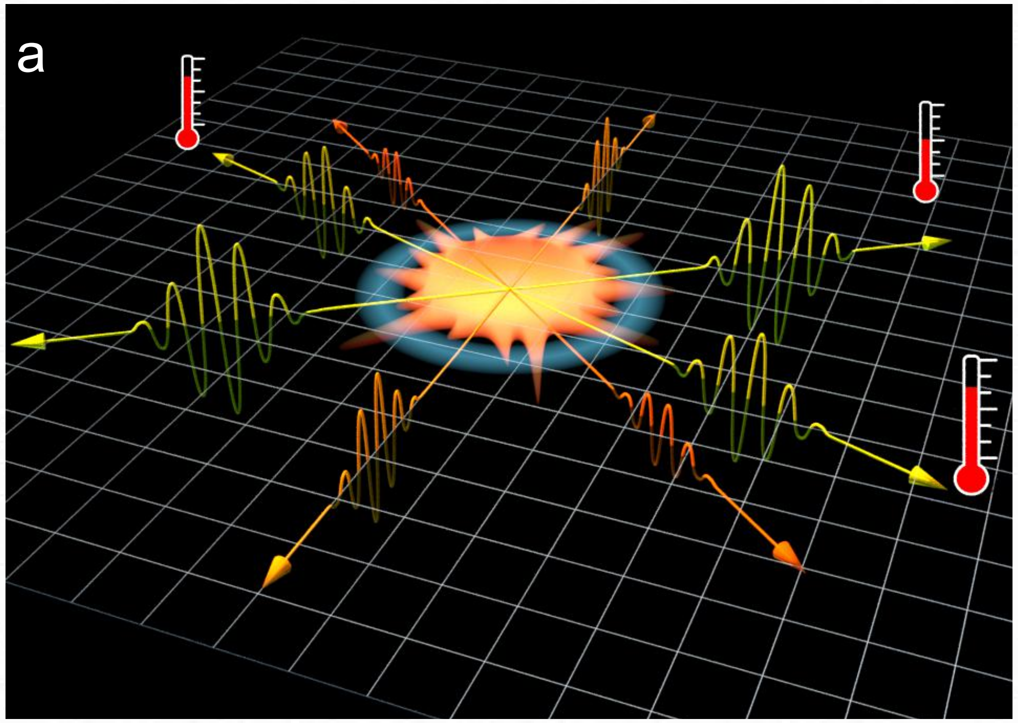
$$\beta \to 0 \qquad |\Psi\rangle_{TFD} \to \sum_n |n\rangle_L |n\rangle_R$$

Thermofield Double State: Example

$$H = i\hbar g \sum_{|\vec{k}|=k_f} (a_k^\dagger a_{-k}^\dagger - a_k a_{-k})$$

$$|\psi(\tau)\rangle = e^{-ih\tau/\hbar}|0\rangle = \frac{1}{\cosh(g\tau)} \sum_{n=0}^{\infty} \tanh^n(g\tau)|n, n\rangle$$

Long time limit $\longrightarrow$ **TFD**



J. Hu. et.al. Nat. Phys. 2018

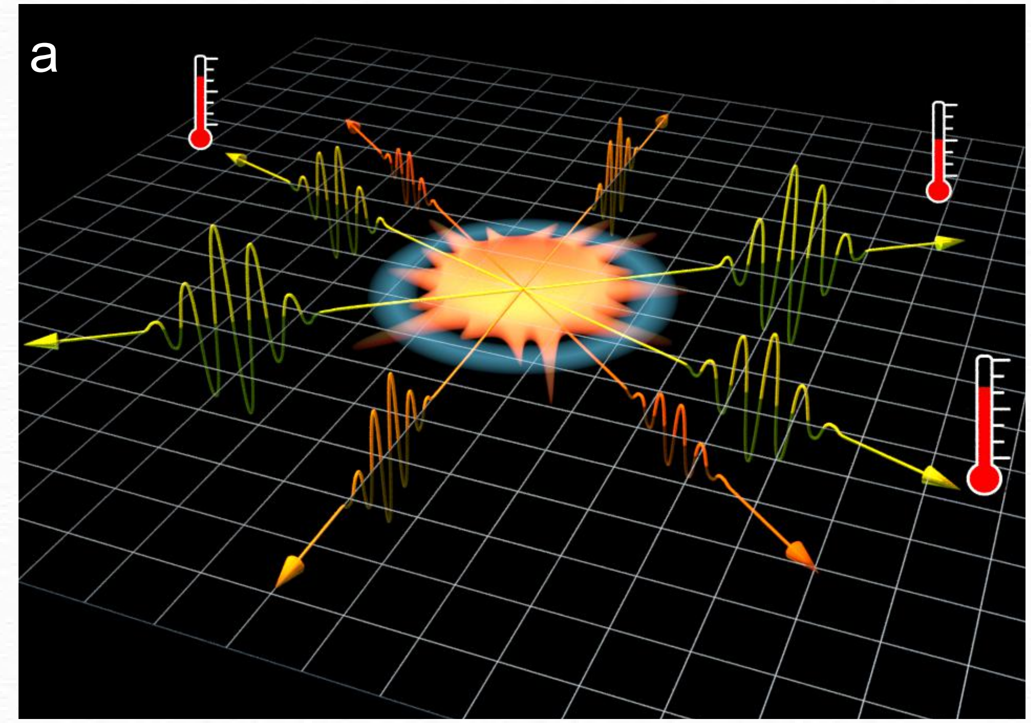
Thermofield Double State: Example

$$H = i\hbar g \sum_{|\vec{k}|=k_f} (a_k^\dagger a_{-k}^\dagger - a_k a_{-k})$$

$$|\psi(\tau)\rangle = e^{-ih\tau/\hbar}|0\rangle = \frac{1}{\cosh(g\tau)} \sum_{n=0}^{\infty} \tanh^n(g\tau)|n,n\rangle$$

$\xrightarrow{\hspace{3em}}$ **TFD**

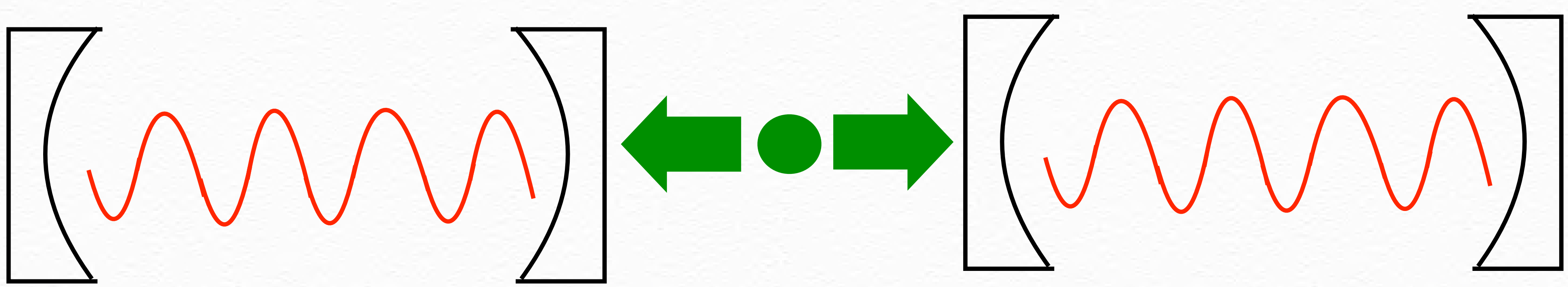
Long time limit



J. Hu. et.al. Nat. Phys. 2018

Two-Mode Squeezed State:

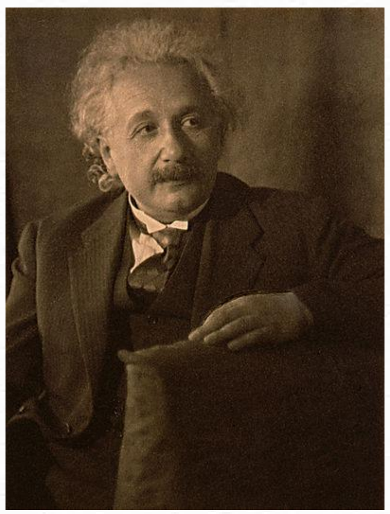
$$\hat{H} = \hat{a}_L^\dagger \hat{a}_R^\dagger + \hat{a}_L \hat{a}_R$$

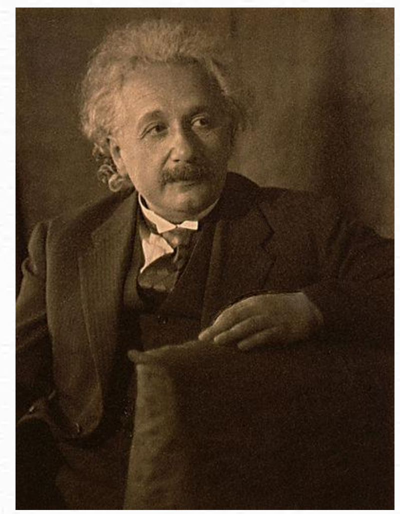


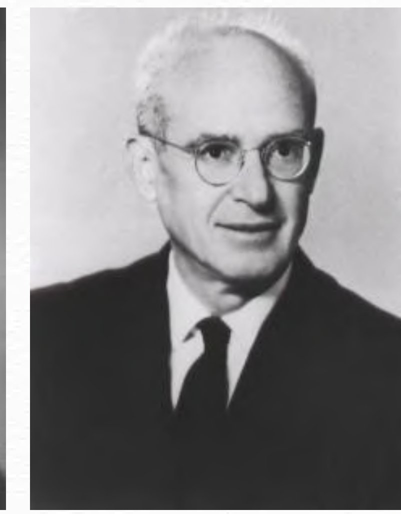
L **R**

ER=EPR Conjecture

ER = EPR







Einstein Rosen Einstein Podolsky Rosen

Einstein-Rosen Bridge

Wormhole

Thermofield Double State

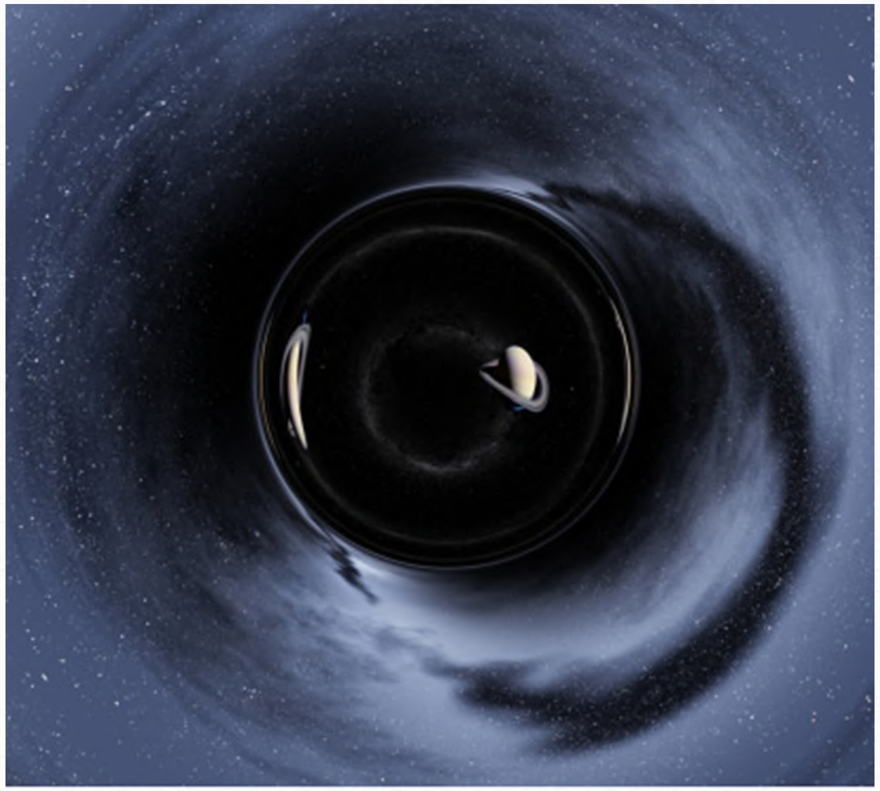
Quantum Entanglement

"=" best understood in term of holographic duality

Maldacena and Susskind, 2013

Wormhole

The movie " Interstellar " 星际穿越



Visualizing *Interstellar*'s Wormhole

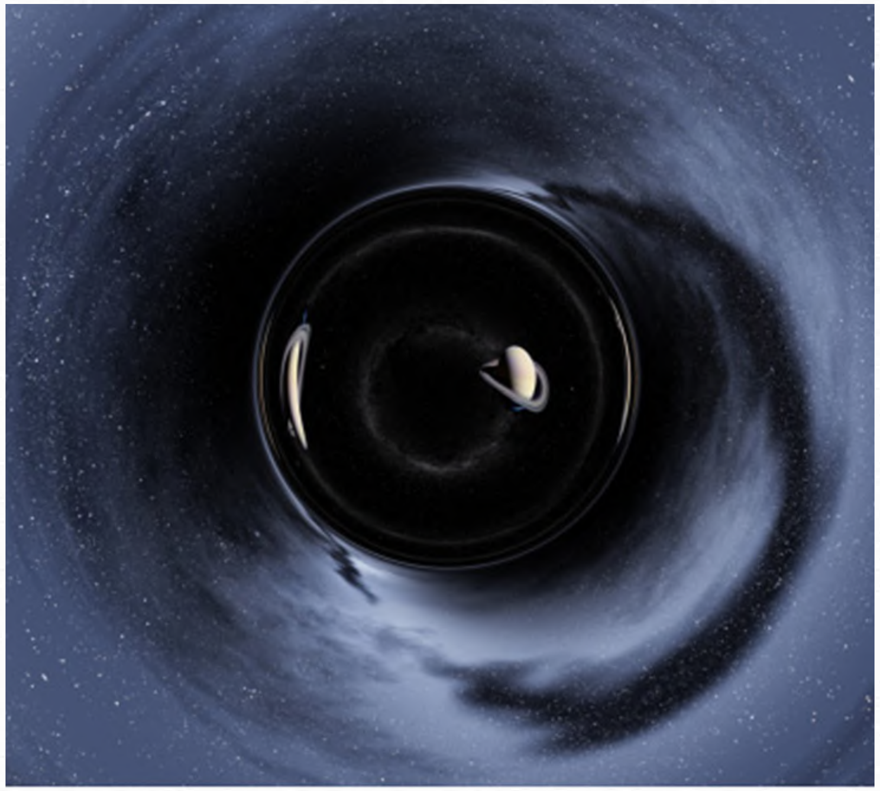
Oliver James, Eugénie von Tunzelmann, Paul Franklin, and Kip S. Thorne

Citation: American Journal of Physics **83**, 486 (2015); doi: 10.1119/1.4916949

arXiv: 1502.03809

Wormhole

The movie “ Interstellar ” 星际穿越



- The Wormhole in “Interstellar” is traversable
- The Einstein-Rosen Bridge is NOT traversable

What this is all about ?

Hayden and Preskill ask:

Can one retrieval information from a black hole ?

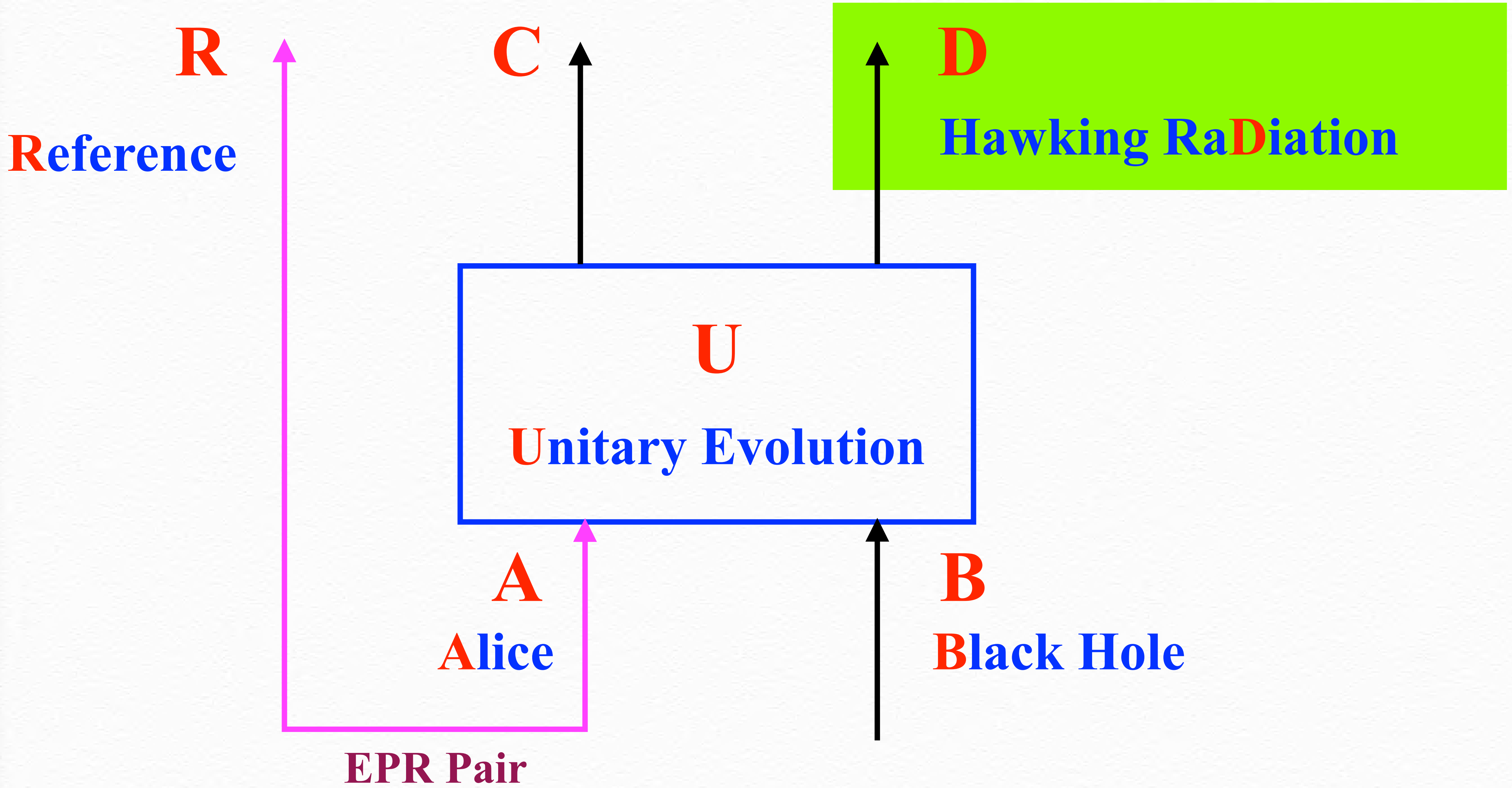
=

Can one retrieval initial state information
when a quantum system thermalizes

- **Information scrambling in quantum thermalization prevents this**
- **The more complicated a quantum system,**
  the faster information scrambles
- **Thermofield Double State can help !**

Hayden-Preskill Protocol

$$\mathcal{H}_B \gg \mathcal{H}_D \gg \mathcal{H}_A$$

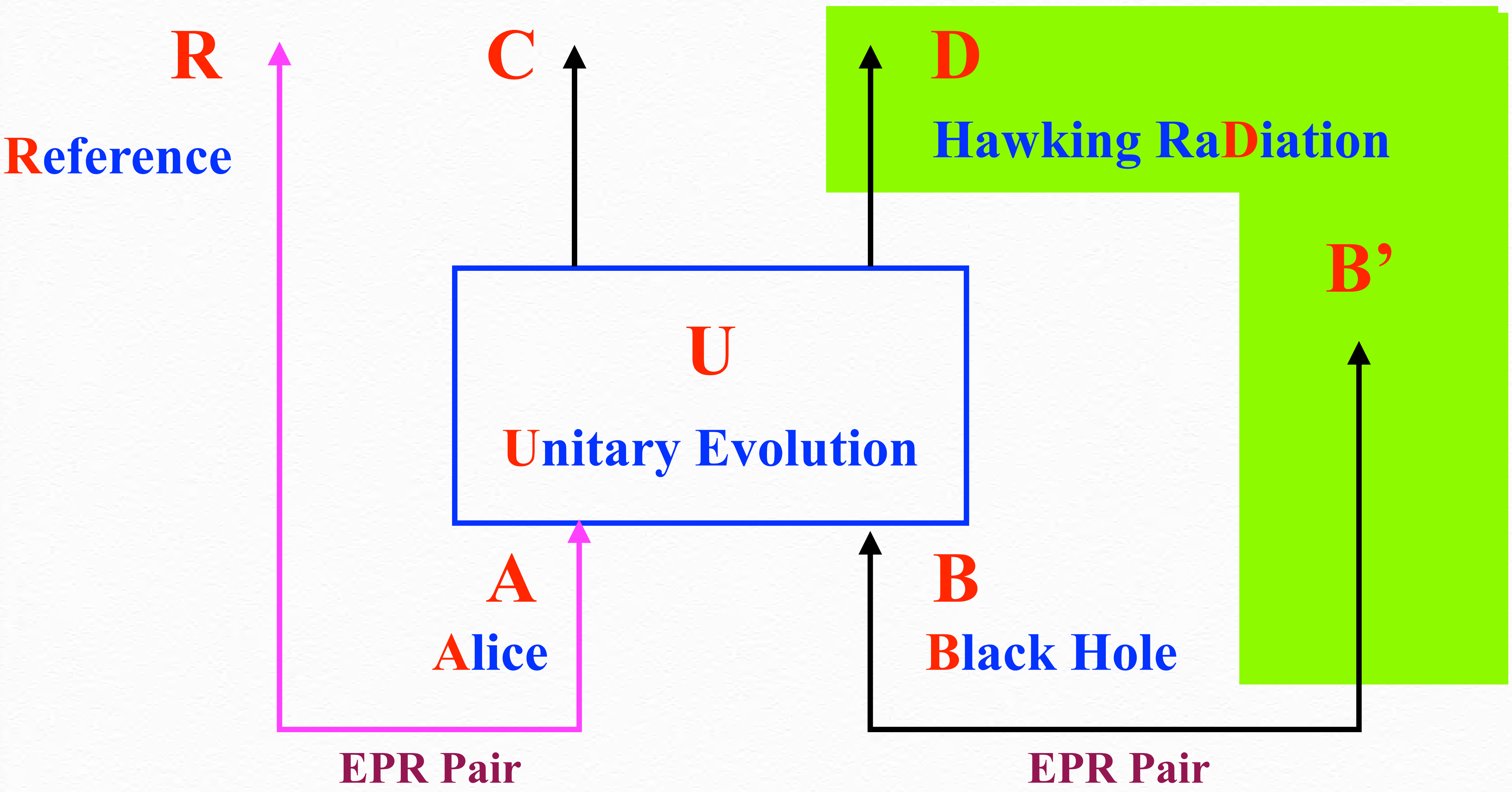


$$I^{(2)}(R, D) = 0$$

Hayden, Priskill, 2007

Hayden-Preskill Protocol

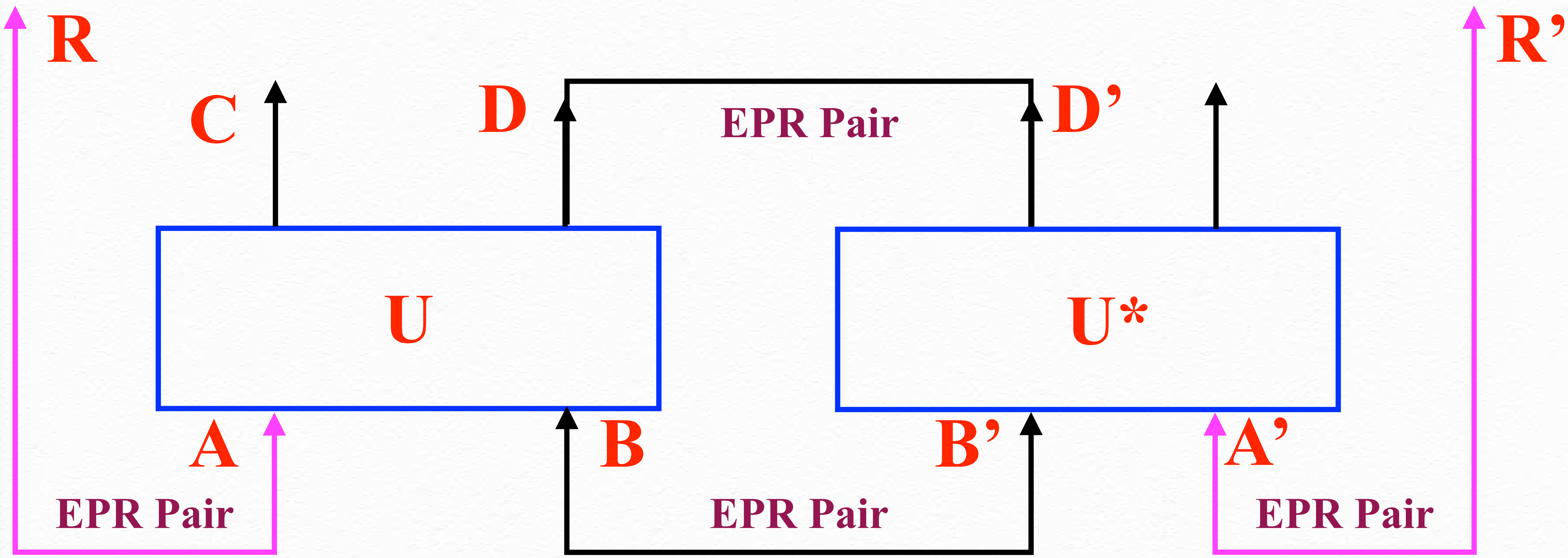
$$\mathcal{H}_B \gg \mathcal{H}_D \gg \mathcal{H}_A$$



$$I^2(R, DB') = 2\log d_R$$

Hayden, Priskill, 2007

Hayden-Preskill Protocol: Measurement-Based

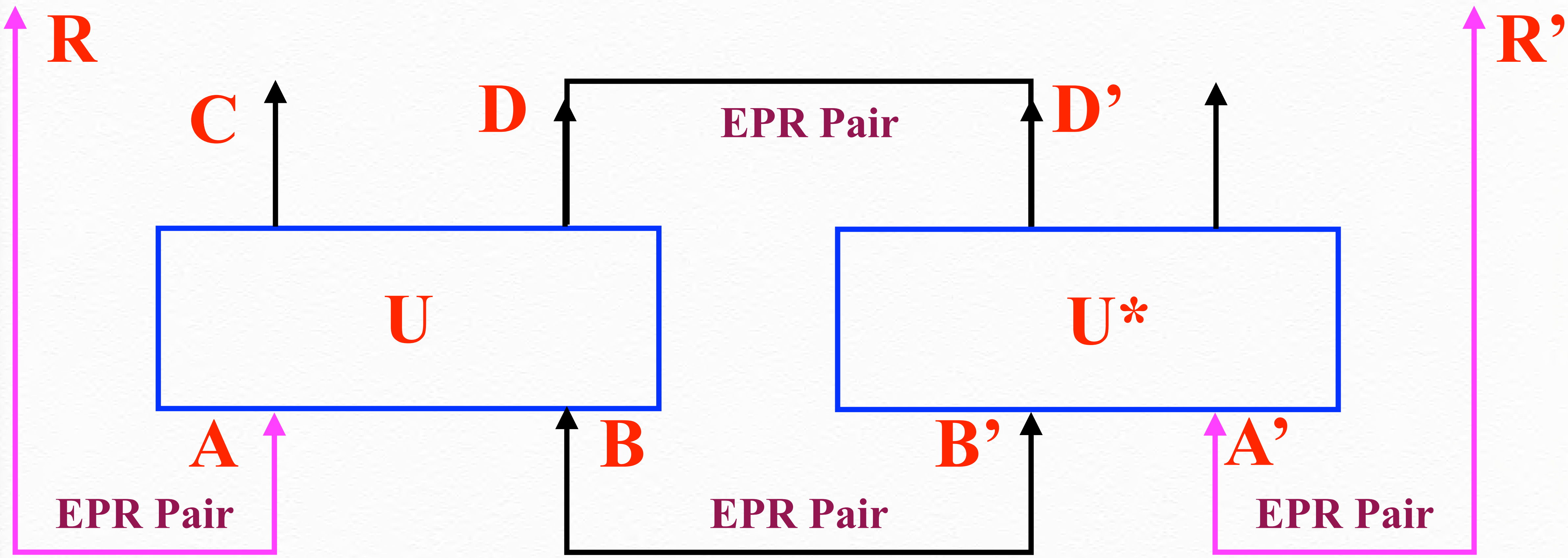


- Fully scrambled (black hole type dynamics)
- Two identical copy of the Hamiltonian (up to a minus sign)

$$\boxed{P(RR'|DD') = 1}$$

Yoshida, Kitaev, 2017

Physical Realization

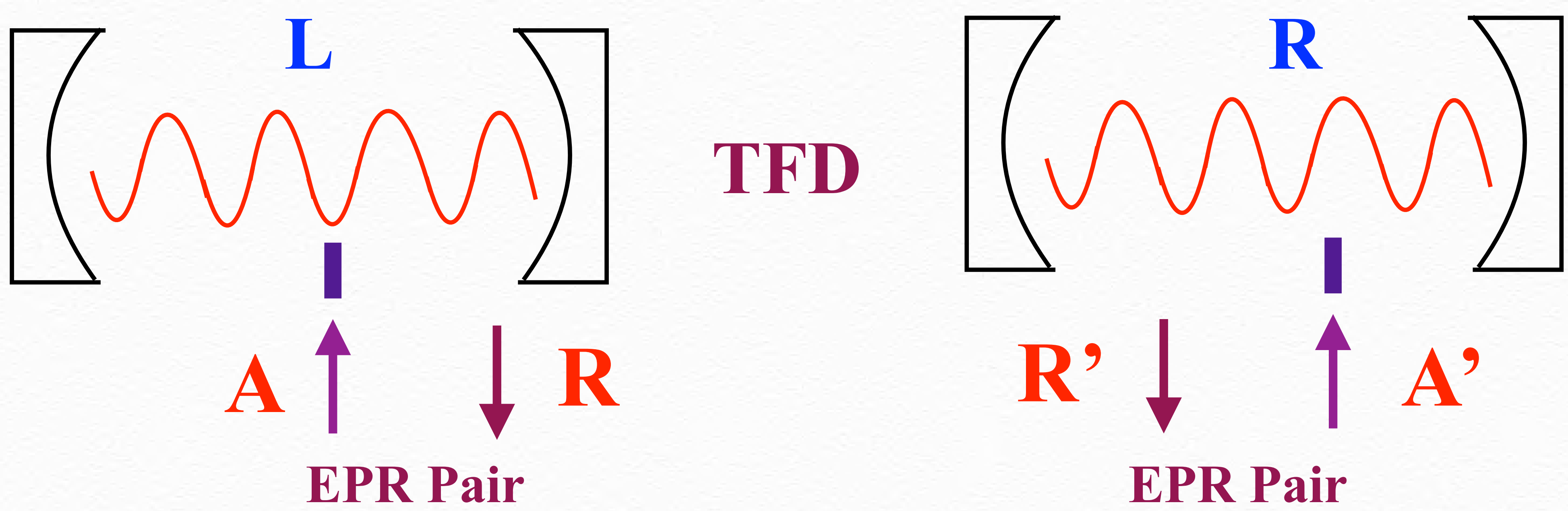


- Fully scrambled (black hole type dynamics) ?
- Two identical copy of the Hamiltonian (up to a minus sign) ?

$$\boxed{P(RR'|DD')=1}$$?

Physical Realization

$H_L = \hbar\omega_0 \hat{a}^\dagger \hat{a} +$ $+ \hbar\omega_z \sigma_z$ $\qquad$ $H_R = -\hbar\omega_0 \hat{a}^\dagger \hat{a}$ $- \hbar\omega_z \sigma_z$

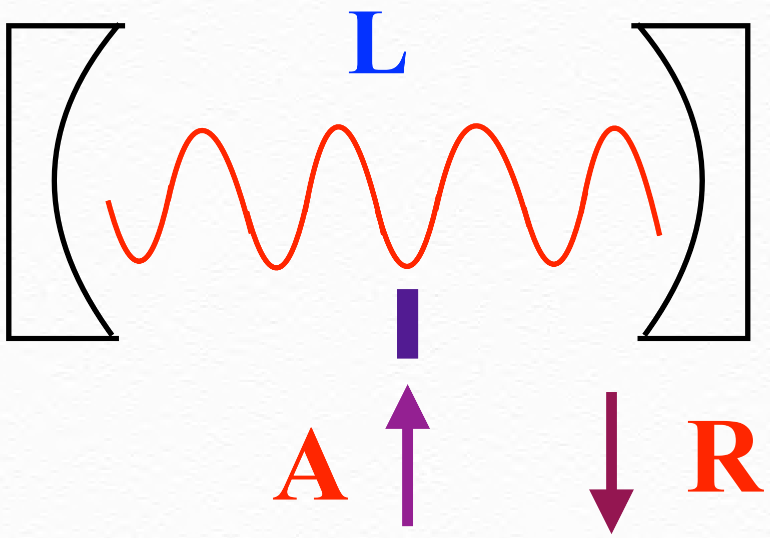


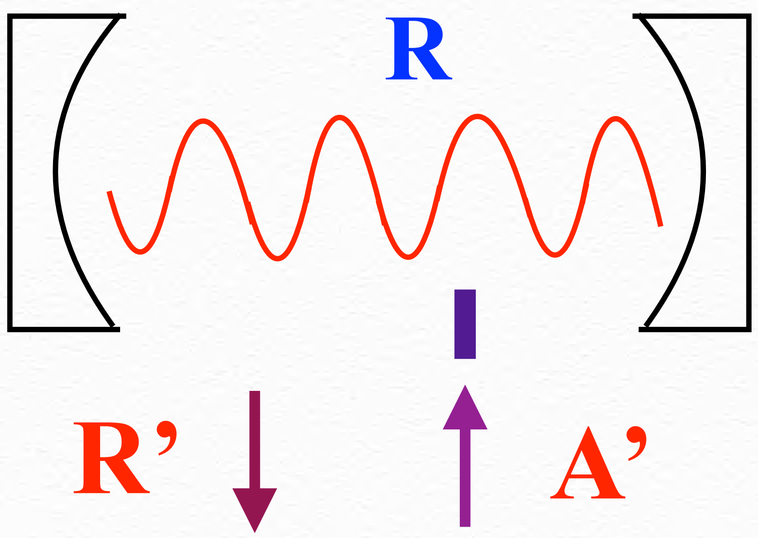
- **Initial State Preparation**

Physical Realization

$H_L = \hbar\omega_0 \hat{a}^\dagger \hat{a} + g(\hat{a}^\dagger + \hat{a})\sigma_x + \hbar\omega_z\sigma_z$

$H_R = -\hbar\omega_0 \hat{a}^\dagger \hat{a} - g(\hat{a}^\dagger + \hat{a})\sigma_x - \hbar\omega_z\sigma_z$



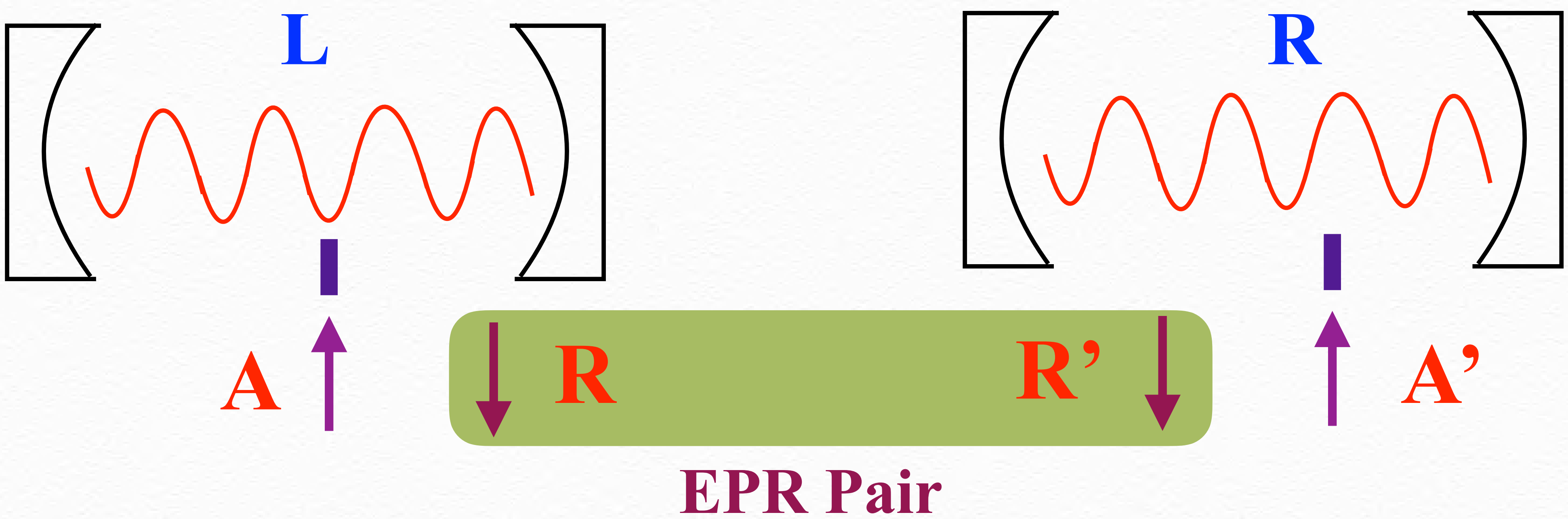


- **Initial State Preparation**
- **Turn on coupling and let the system evolve until scrambling**

$$\rho^{\mathrm{i}}_{\mathrm{RA}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \rho^{\mathrm{f}}_{\mathrm{RA}} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Physical Realization

$$H_L = \hbar\omega_0 \hat{a}^\dagger \hat{a} + g(\hat{a}^\dagger + \hat{a})\sigma_x + \hbar\omega_z\sigma_z \qquad H_R = -\hbar\omega_0 \hat{a}^\dagger \hat{a} - g(\hat{a}^\dagger + \hat{a})\sigma_x - \hbar\omega_z\sigma_z$$

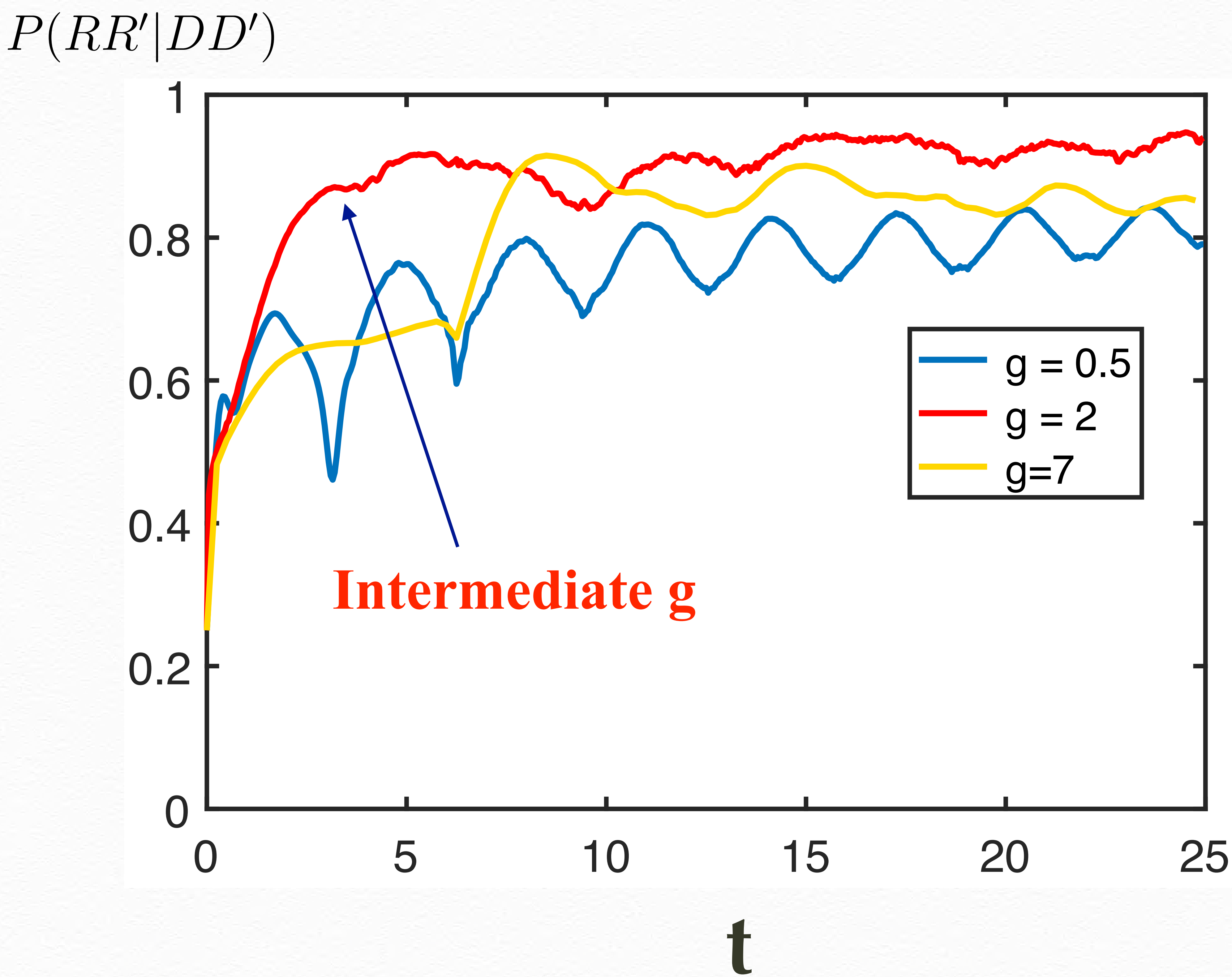


- **Initial State Preparation**
- **Turn on coupling and let the system evolve until scrambling**
- **Projected into EPR state of D and D'**

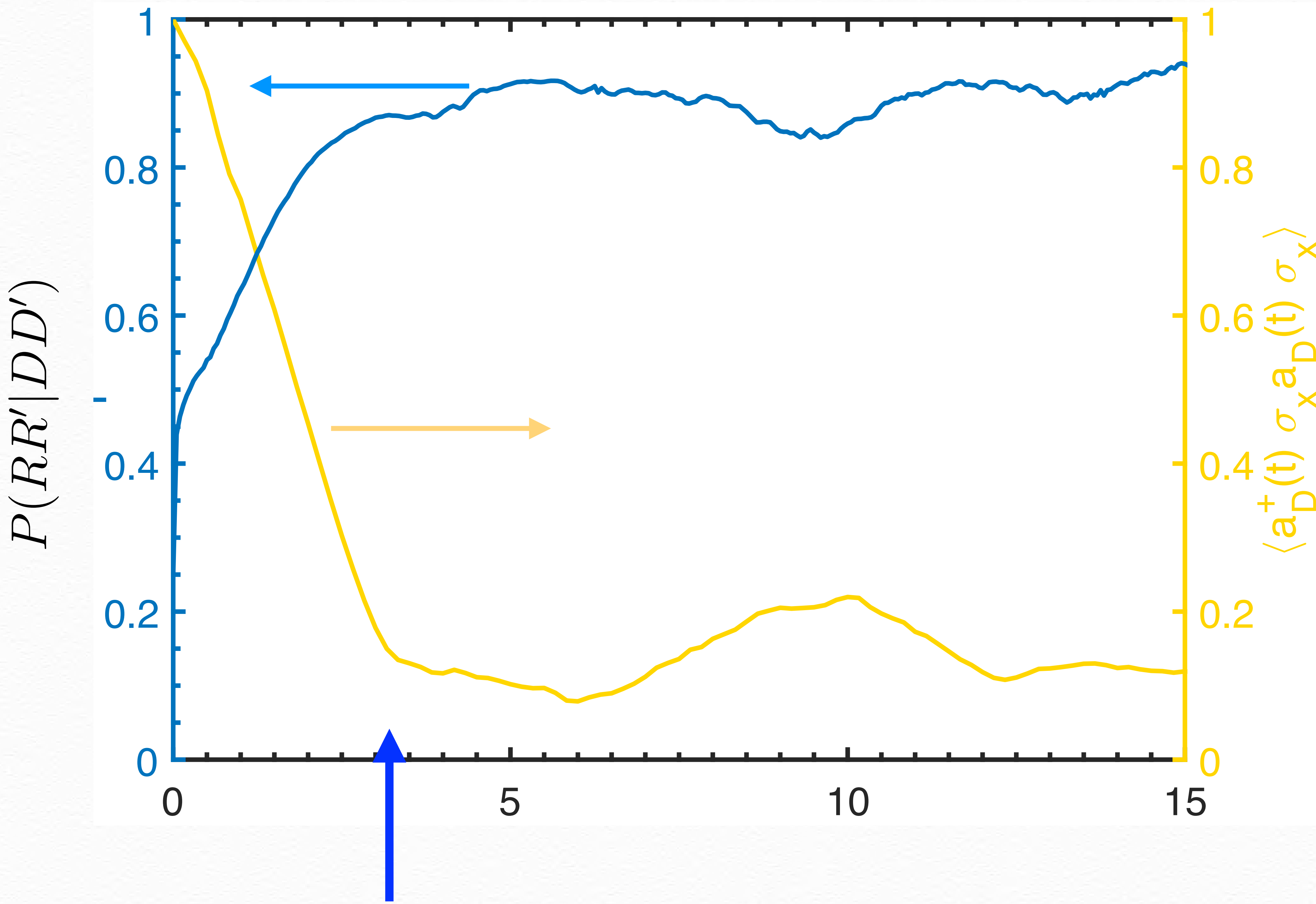
$$|DD'\rangle_{EPR} = \sum_{n=1}^{n_D} |n_L n_R\rangle$$

$$\mathcal{P}_{DD'} = |DD'\rangle\langle DD'|$$

Decoding Efficiency v.s. Coupling Constant



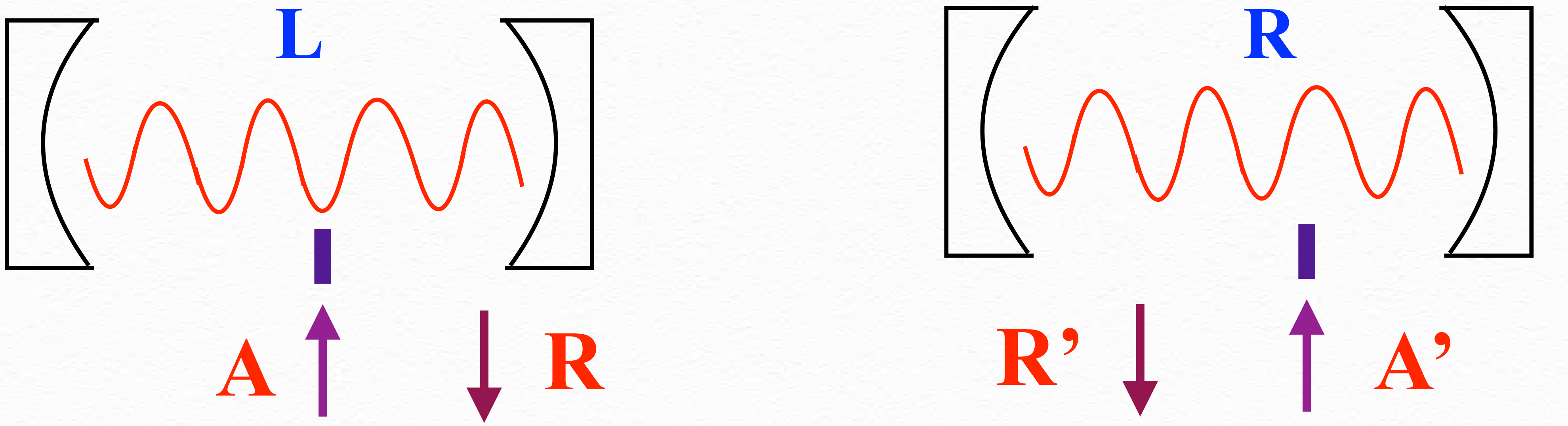
Decoding v.s. Scrambling

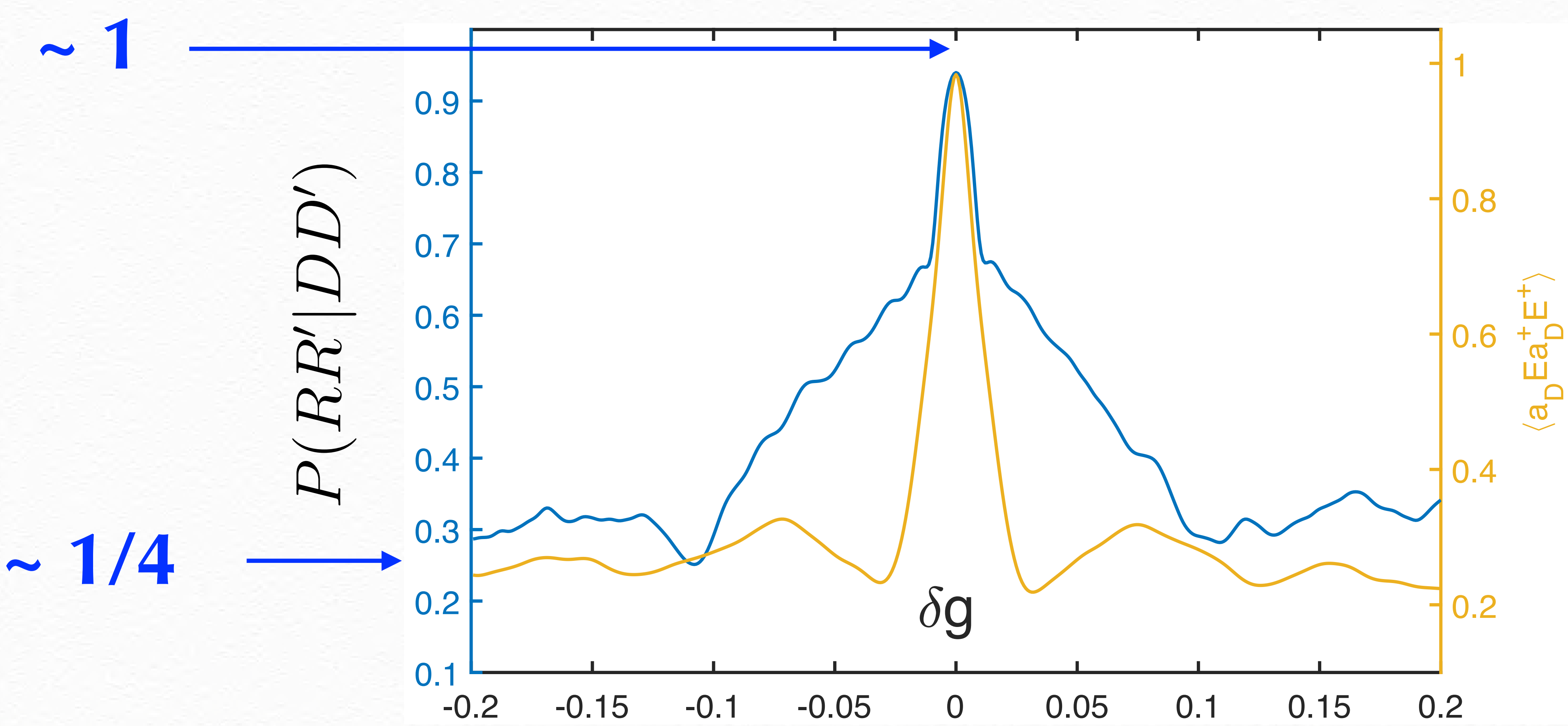


Scrambling time

Stability of the decoding protocol

$$H_L = \hbar\omega_0 \hat{a}^\dagger \hat{a} + g(\hat{a}^\dagger + \hat{a})\sigma_x + \hbar\omega_z\sigma_z \qquad H_R = -\hbar\omega_0' \hat{a}^\dagger \hat{a} - g'(\hat{a}^\dagger + \hat{a})\sigma_x - \hbar\omega_z'\sigma_z$$





Stability of the decoding protocol

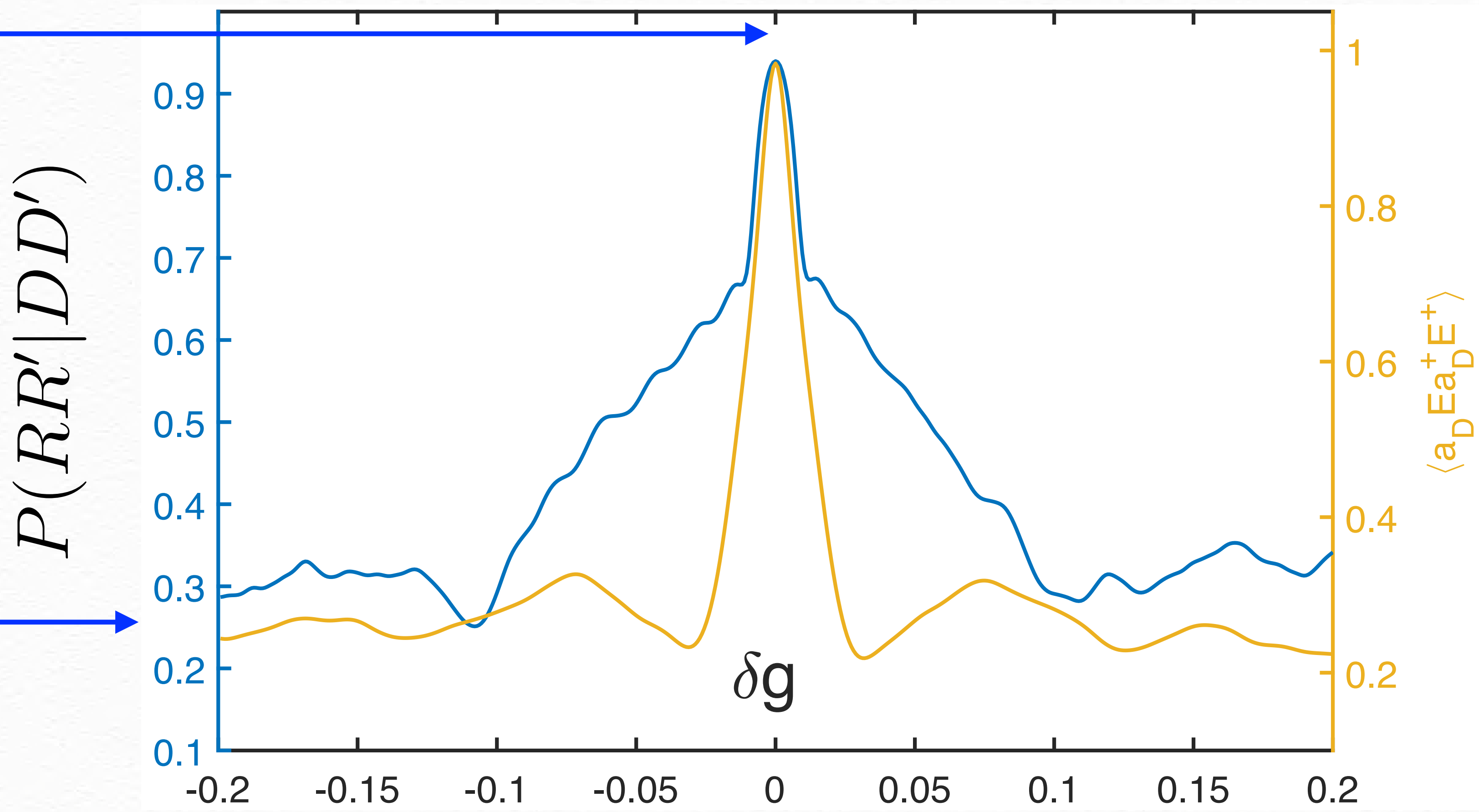
Exact Relation:

$$P(RR'|DD') = \frac{\sum_{O_D \subset P_D} \langle O_D E O_D^\dagger E^\dagger \rangle}{d_A^2 - 1 + \sum_{O_D \subset P_D} \langle O_D E O_D^\dagger E^\dagger \rangle}$$

$$\hat{E} = \hat{U}\hat{U}'$$

~ 1

~ 1/4



Stability of the decoding protocol

Exact Relation:

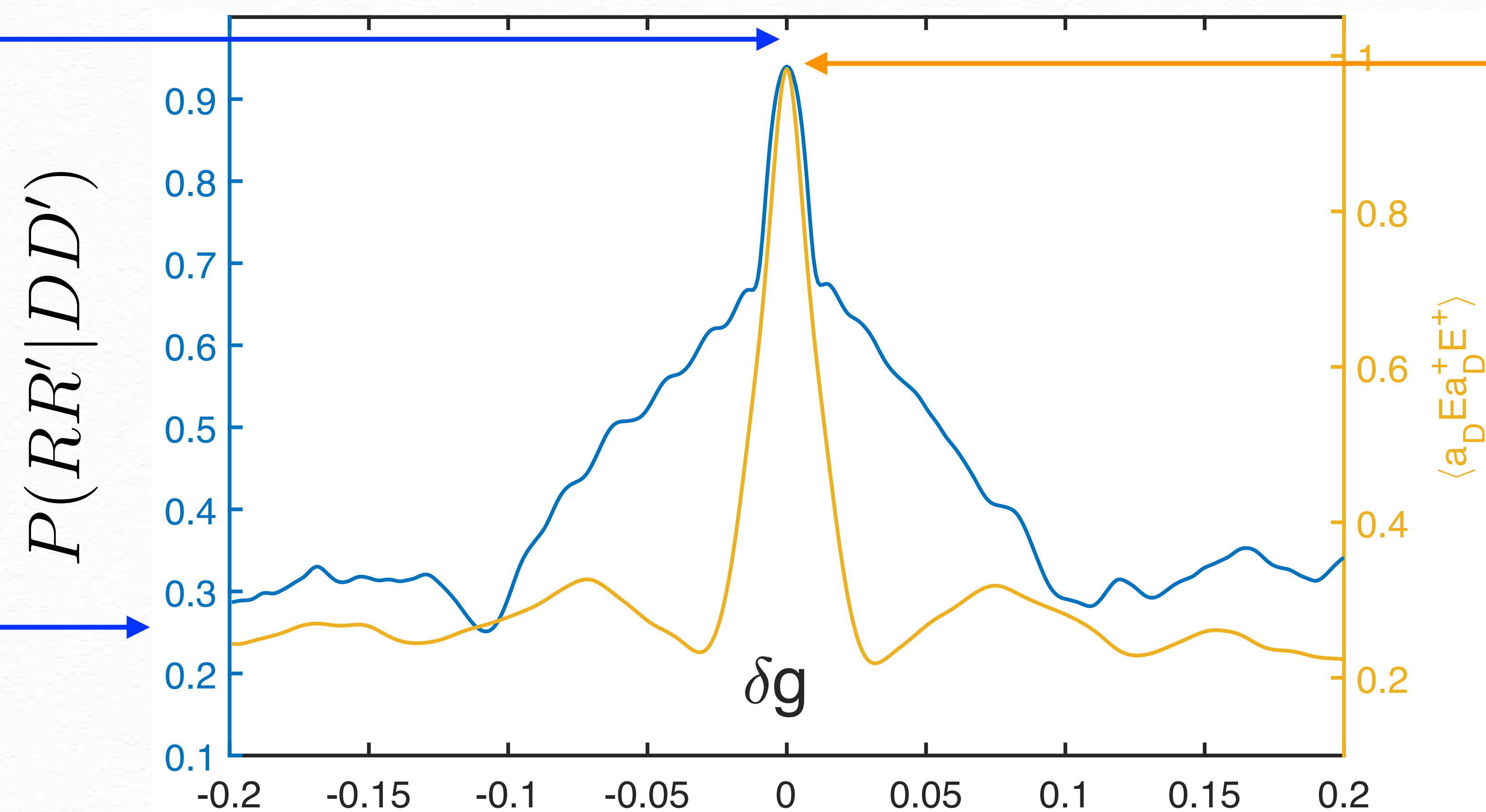
$$\delta g \to 0 \qquad \hat{E} \to \hat{I}$$

$$P(RR'|DD') = \frac{\sum_{O_D \subset P_D} \langle O_D E O_D^\dagger E^\dagger \rangle}{d_A^2 - 1 + \sum_{O_D \subset P_D} \langle O_D E O_D^\dagger E^\dagger \rangle} \rightarrow \frac{d_D^2}{d_A^2 + d_D^2 - 1} \sim 1$$

$$\hat{E} = \hat{U}\hat{U}'$$

~ 1

~ 1



~ 1/4

Stability of the decoding protocol

Exact Relation:

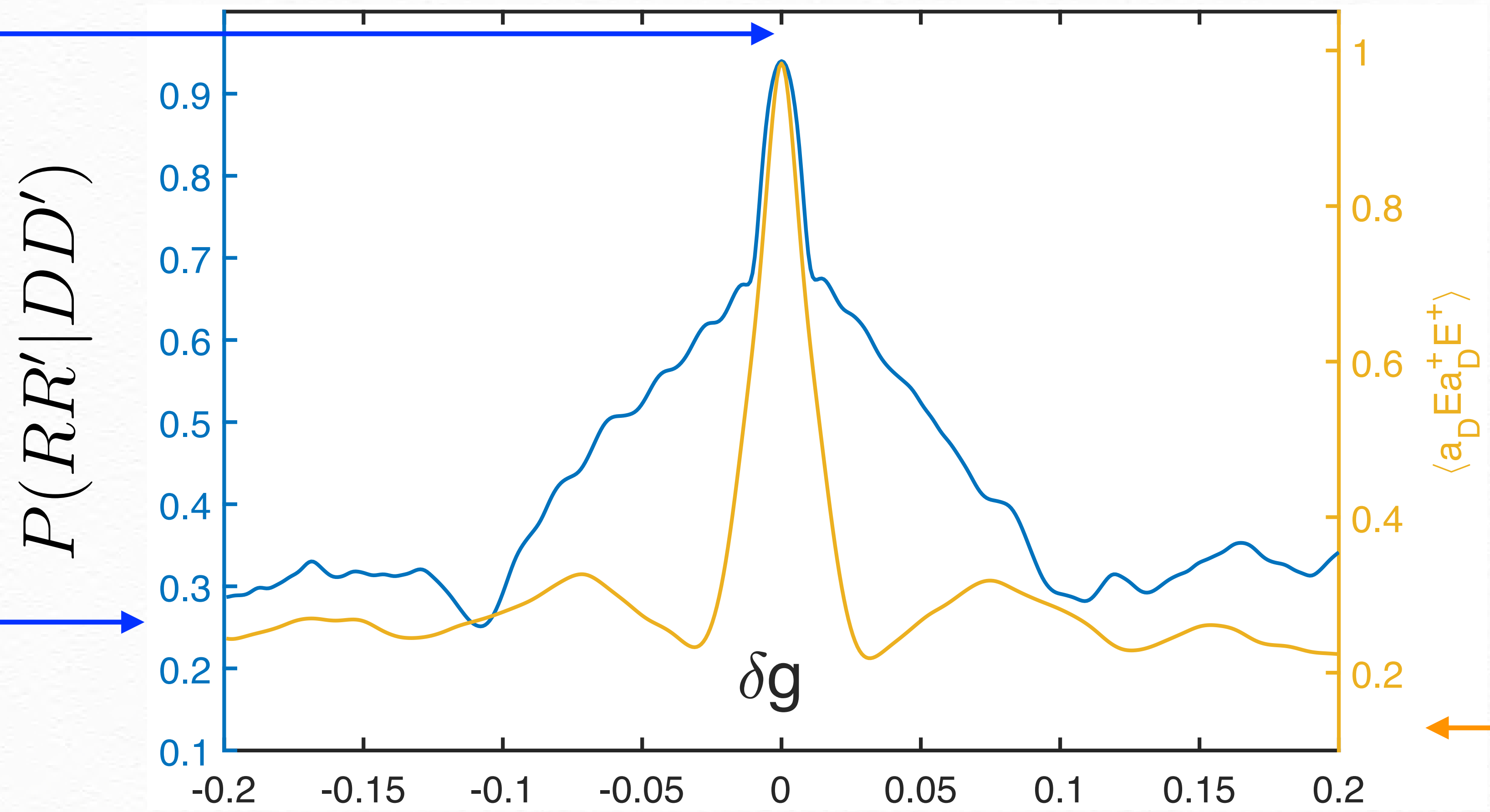
δg **large;** $\hat{E} \to$ **random**

$$P(RR'|DD') = \frac{\sum_{O_D \subset P_D} \langle O_D E O_D^\dagger E^\dagger \rangle \longrightarrow \mathbf{1}}{d_A^2 - 1 + \sum_{O_D \subset P_D} \langle O_D E O_D^\dagger E^\dagger \rangle} \longrightarrow \frac{1}{d_A^2}$$

$$\hat{E} = \hat{U}\hat{U}'$$

~ 1

~ 1/4



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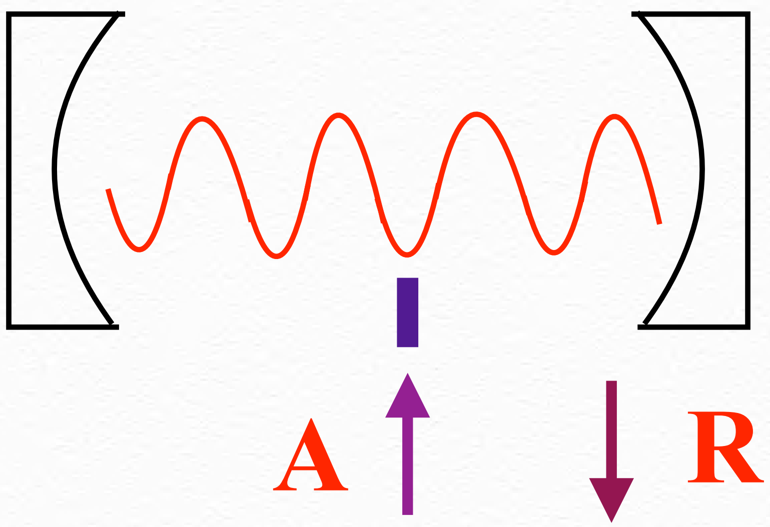
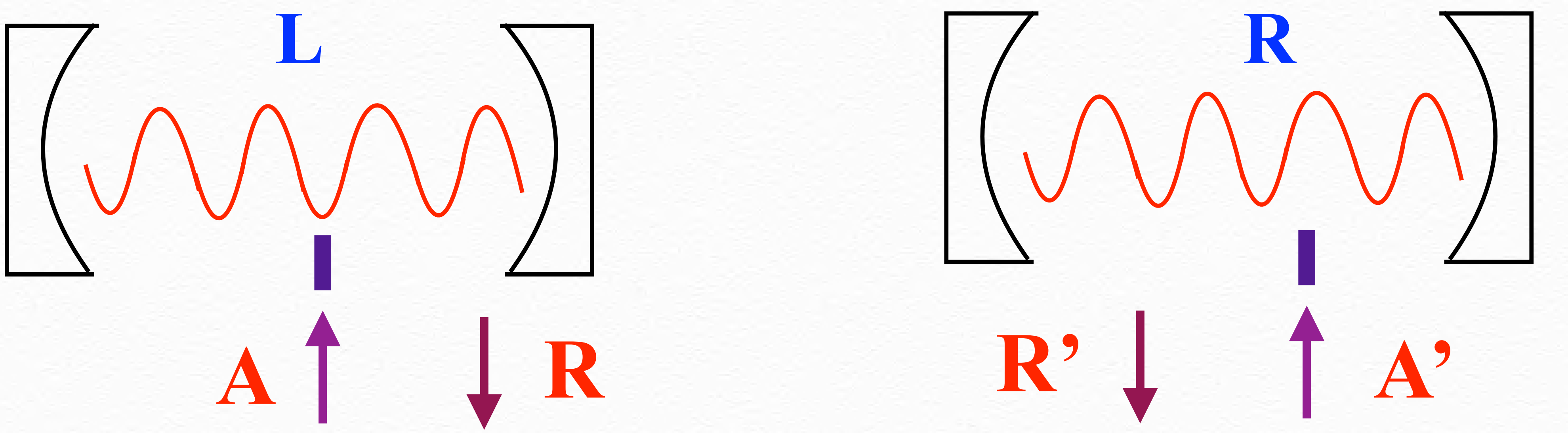
Summary



败也萧何

Because of *information scrambling*, we can not decode the initial state information for a single system

Take Home Message



败也萧何

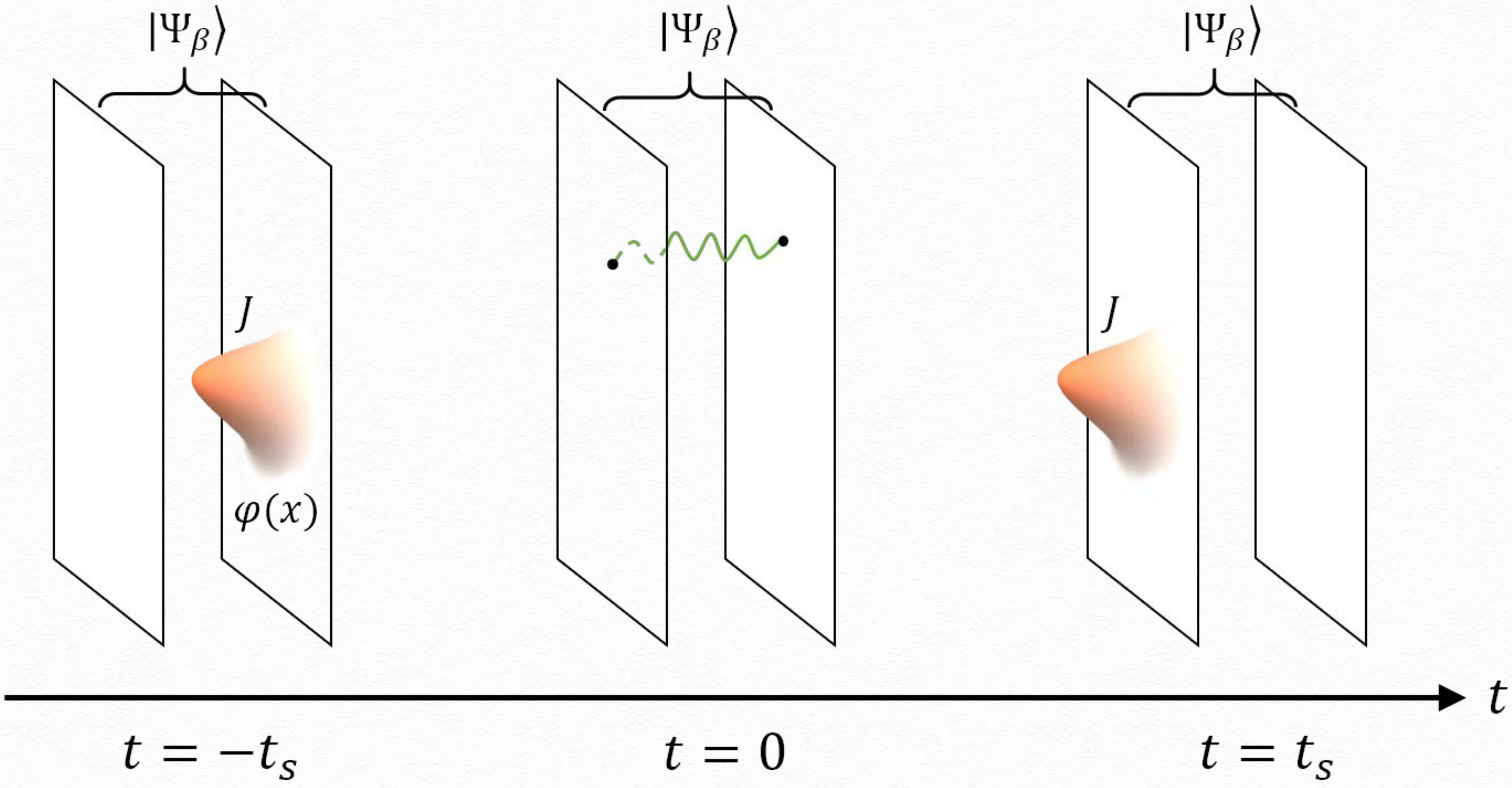
Because of *information scrambling*, we can not decode the initial state information for a single system

成也萧何

Thank to *information scrambling*, we can decode the initial state information for a thermofield double system

Outlook: Traversable Wormhole

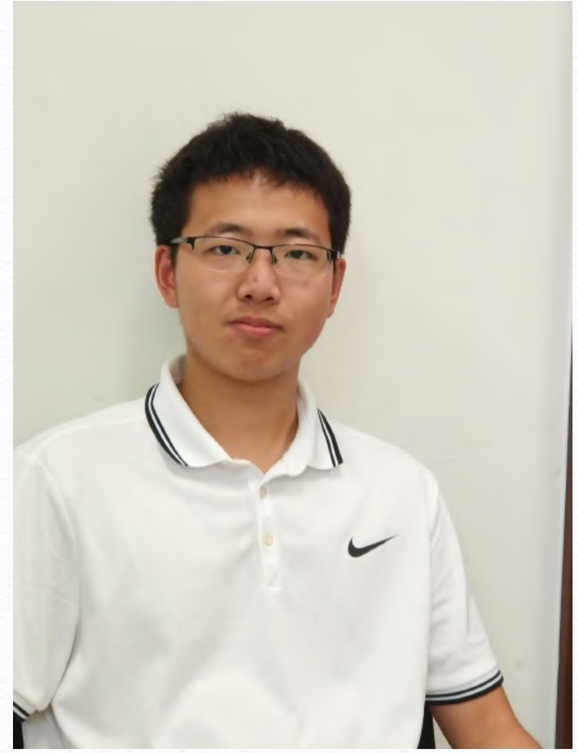
How to make a wormhole traversable ?



To be continued …

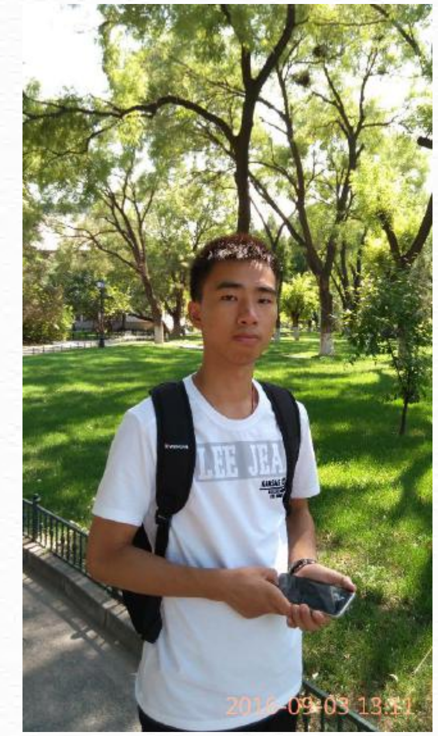
Gao, Jafferies and Wall, 2017; Maldacena, Stanford and Yang, 2017
Ping Gao and Hong Liu, 2018

Yanting Cheng
程艳婷



Chang Liu

刘畅



Jinkang Guo

郭金康

Yu Chen 陈宇

Pengfei Zhang 张鹏飞

Thank You Very Much！